Detuning errors in SC resonators

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Introduction

Accelerating cavities are typically detuned from their resonant frequency in order to minimise the power required to maintain the accelerating voltage in the presence of loading by a beam with a non-zero synchronous phase. An incorrectly detuned cavity will require additional power, which is subsequently reflected back to the RF source, and thus wasted. This note calculates the additional power required due to such detuning errors.

1 Optimal power

For a required cavity voltage, V_{cav}, beam coupling, (R/Q), loaded quality factor, Q_L, beam current, I_{b0}, synchronous phase, ϕ_b, a RF frequency, f, and a detuning of Δf, the required forward power from the generator, P_G, can be shown to be the following.

$$P_G = \frac{1}{4}\frac{V_{cav}^2}{\left(\frac{R}{Q}\right)Q_L}\left[\left(1+\frac{\left(\frac{R}{Q}\right)Q_L I_{b0}}{V_{cav}}\cos\phi_b\right)^2+\left(2Q_L\frac{\Delta f}{f}+\frac{\left(\frac{R}{Q}\right)Q_L I_{b0}}{V_{cav}}\sin\phi_b\right)^2\right] \tag{1}$$

Holding all other parameters constant, it can be seen that the optimal detuning, Δf_{opt}, is achieved when the second term in equation 1 goes to zero.

$$2Q_L\frac{\Delta f_{opt}}{f} = -\frac{\left(\frac{R}{Q}\right)Q_L I_{b0}}{V_{cav}}\sin\phi_b \tag{2}$$

For this optimal detuning, the optimal forward power, P_{Gopt}, is as follows.

$$P_{Gopt} = \frac{1}{4}\frac{V_{cav}^2}{\left(\frac{R}{Q}\right)Q_L}\left(1+\frac{\left(\frac{R}{Q}\right)Q_L I_{b0}}{V_{cav}}\cos\phi_b\right)^2 \tag{3}$$

2 Power for a detuning error

In the case of a detuning error, Δf_{err}, where the total detuning of the cavity is, $\Delta f = \Delta f_{opt} + \Delta f_{err}$, the forward power, P_{Gerr}, required is as follows.

$$P_{Gerr} = \frac{1}{4}\frac{V_{cav}^2}{\left(\frac{R}{Q}\right)Q_L}\left[\left(1+\frac{\left(\frac{R}{Q}\right)Q_L I_{b0}}{V_{cav}}\cos\phi_b\right)^2+\left(2Q_L\frac{\Delta f_{err}}{f}\right)^2\right] \tag{4}$$

Expressed as a ratio, this is as follows.

$$\frac{P_{Gerr}}{P_{Gopt}} = 1+\frac{\left(2Q_L\frac{\Delta f_{err}}{f}\right)^2}{\left(1+\frac{\left(\frac{R}{Q}\right)Q_L I_{b0}}{V_{cav}}\cos\phi_b\right)^2} \tag{5}$$

2.1 Assuming optimal Q_L

In the situation where the loaded quality factor of the cavity has been chosen optimally, the power ratio can be written as follows.

$$\frac{P_{Gerr}}{P_{Gopt}} = 1 + \left(Q_L \frac{\Delta f_{err}}{f} \right)^2 \tag{6}$$

If it was decided that the maximum additional forward power required due to a detuning error should be limited to 1% of the optimal forward power, this would require a maximum detuning error, $\Delta f_{err,max}$,

$$\Delta f_{err,max} = 0.1 \cdot \frac{f}{Q_L} \tag{7}$$