Loaded Q Calculations for the ESS Superconducting RF Linac

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Introduction

This note uses the formulation described in the note “Transit Time Equivalent Circuit for a Superconducting RF Cavity”, ESS Docs Document 294-v1 to calculate the loaded quality factor for spoke cavities and the medium beta cavities of the ESS linac. The lattice data is from the OPTIMUS lattice released on 26-July-2013. The beam current assumed is 62.5 mA.

Formulas

The voltage transit time is calculated from the field map provided by the cavity designers with the following integral:

$$VT_{fm}(\beta) = \int E_{zfm}(z) \begin{pmatrix} cos\left(2\pi \frac{z}{\beta\lambda}\right) \\ sin\left(2\pi \frac{z}{\beta\lambda}\right) \end{pmatrix} dz \qquad (1)$$

where β is v/c, λ is the wavelength, and E_{zfm} is the longitudinal component of electrical field as a function of longitudinal position in the cavity for a given stored energy in the cavity W_{sfm}. The cosine term is used for field maps with even symmetry or an odd number of cells and the sine term is used for field maps with odd symmetry or an even number of cells. For the ESS Linac, the spokes section are double spoke cavities that have even symmetry and the medium beta section has six elliptical cells that have an odd symmetry. The ratio of transit times though the cavity for different beam velocities is calculated from the field profile:

$$\frac{T(\beta)}{T_o(\beta_o)} = \frac{VT_{fm}(\beta)}{VT_{fm}(\beta_o)} \qquad (2)$$

The accelerating R/Q which is twice the circuit R/Q is given as:

$$\left.\frac{R}{Q}(\beta_o)\right|_{acc} = 2\left.\frac{R}{Q}(\beta_o)\right|_{cir} = 2\frac{1}{2}\frac{\left(VT_{fm}(\beta_o)\right)^2}{2\pi f_{rf} W_{sfm}} \qquad (3)$$

where f_{rf} is the resonant frequency of the cavity. For short bunches, the rf beam current I_{brf} is twice the peak beam current, I_b.

$$I_{brf} = 2I_b \qquad (4)$$

The loaded Q of the cavity required to match a given voltage VT_o, rf beam current I_{brf}, and synchronous phase angle ϕ_s is.

$$Q_L(\beta_o) = \frac{VT_o}{I_{brf} cos(\phi_s)} \frac{2}{\left.\frac{R}{Q}(\beta_o)\right|_{acc}} \qquad (5)$$

The generator impedance as seen by the cavity is given as:

$$n^2 R_g(\beta_o) = \frac{\frac{R}{Q}(\beta_o)\Big|_{acc}}{2} Q_L(\beta_o) \tag{6}$$

To produce a given voltage VT(β), the required generator current as seen by the cavity is:

$$\frac{I_g(\beta,\beta_o)}{n} = \frac{1}{T(\beta)/T_o(\beta_o)} \frac{VT(\beta)}{n^2 R_g(\beta_o)} + \frac{T(\beta)}{T_o(\beta_o)} I_{brf} cos(\phi_s) \tag{5}$$

The forward power from the generator required to produce this generator current is:

$$P_F(\beta,\beta_o) = \frac{1}{2} n^2 R_g(\beta_o) \left(\frac{I_g(\beta,\beta_o)/n}{2} \right)^2 \tag{6}$$

The power supplied to the beam is:

$$P_B(\beta) = \frac{1}{2} VT(\beta) I_{brf} cos(\phi_s) \tag{7}$$

From energy conservation and for a superconducting cavity where the loss in the cavity walls is negligible, the power reflected back to the generator is the difference between the forward power and the power delivered to the beam:

$$P_R(\beta,\beta_o) = P_F(\beta,\beta_o) - P_B(\beta) \tag{8}$$

The amount of cavity detuning required is:

$$tan(\varphi_D) = \frac{n^2 R_g(\beta_o) I_{brf}}{VT(\beta)} \left(\frac{T(\beta)}{T_o(\beta_o)} \right)^2 sin(\phi_s) \tag{9}$$

<u>Calculations</u>

The R/Q, loaded Q, forward and reflected power for the Optimus lattice was calculated for the spoke cavities and the medium beta cavities as shown in Figures 1-11. The operating beam current was 62.5 mA. The operating point to determine the loaded Q for a given family of cavities was chosen for the beam beta in which the R/Q was maximized. For the spoke cavities, the Q_L was set by the operating point at β=0.505 in which the accelerating R/Q was 428 Ohms and the Q_L was 233x103. For the medium beta cavities, the Q_L was set by the operating point at β=0.705 in which the accelerating R/Q was 395 Ohms and the Q_L was 590x103.

Lattice Data

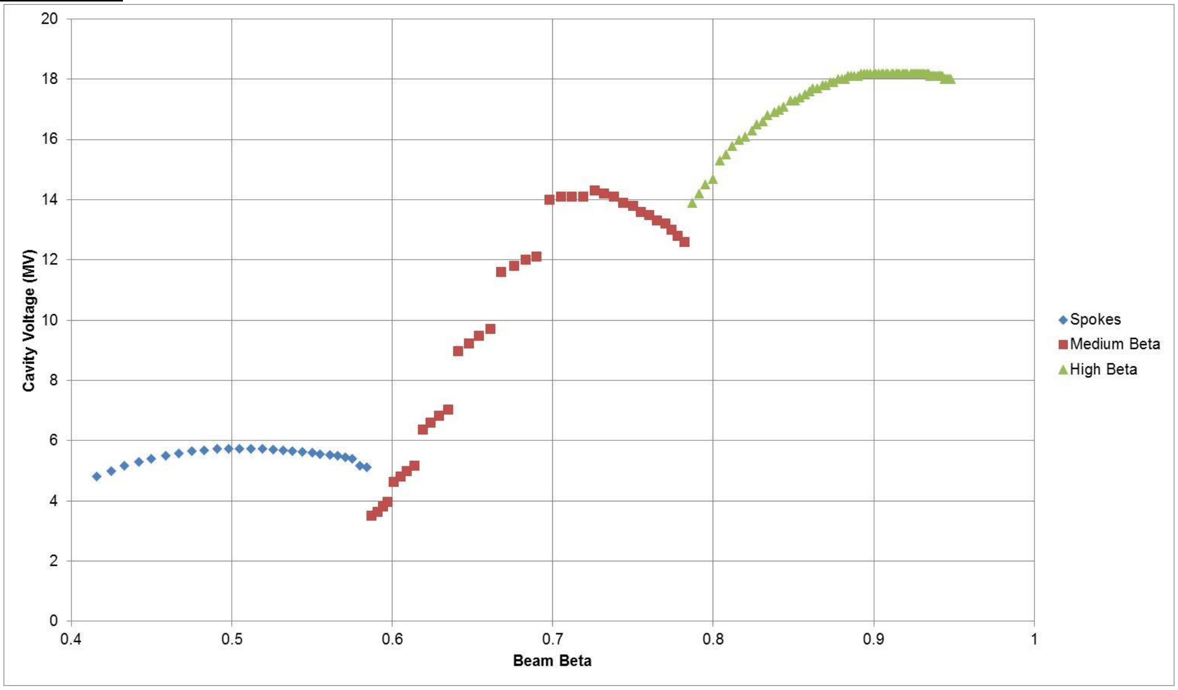

Figure 1. Superconducting Cavity Voltage for the OPTIMUS lattice

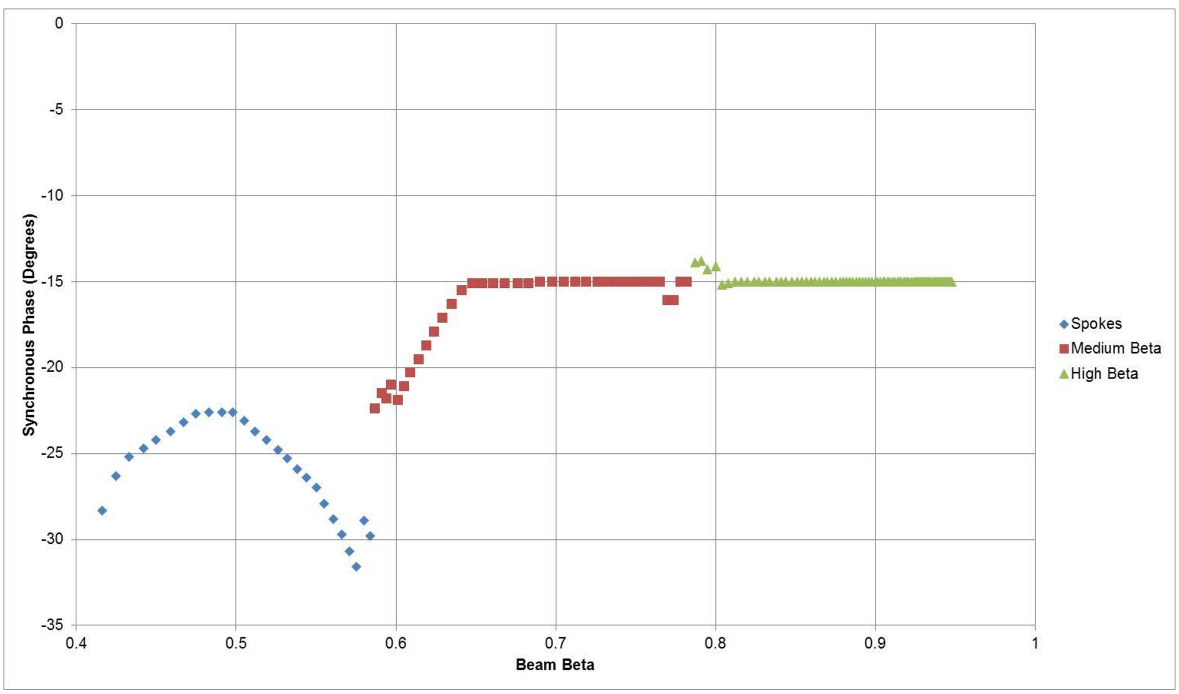

Figure 2. Synchronous Phase for the OPTIMUS lattice

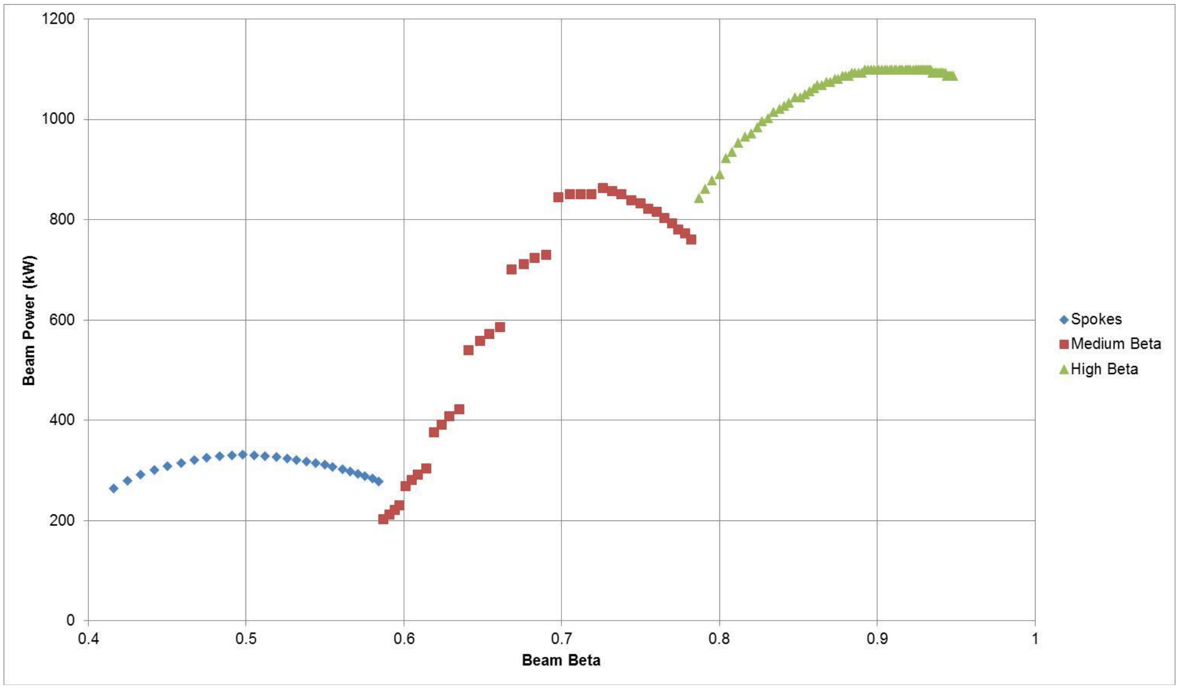

Figure 3. Beam Power for the OPTIMUS lattice

Field Profiles

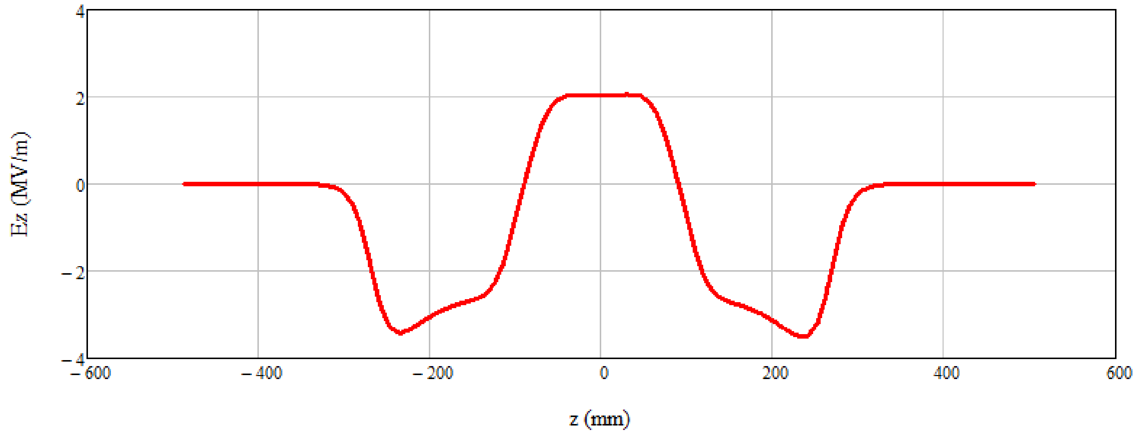

Figure 4. Spoke Field Profile for a stored energy of 1 J

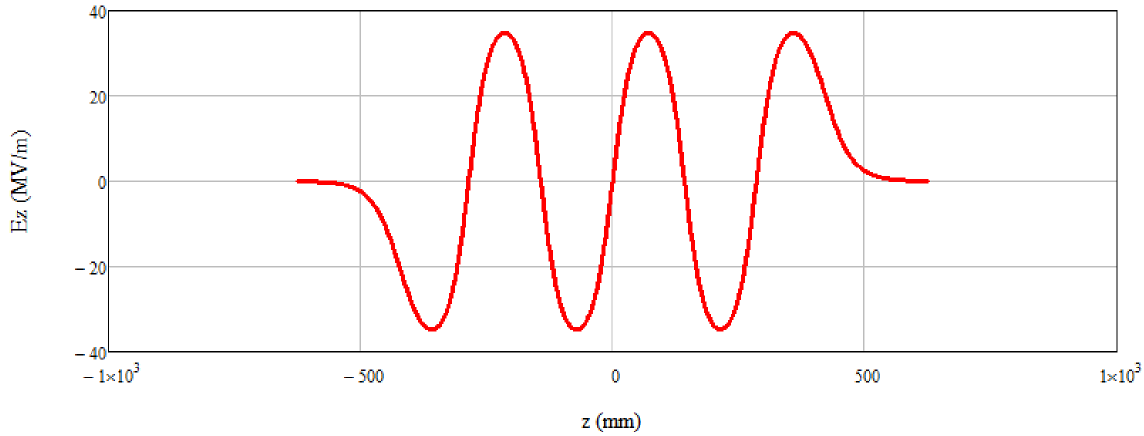

Figure 5. Six cell medium beta field profile with β_g=0.67, a peak surface field of 44 MV/meter, and a stored energy of 144 J.

R/Q Plots

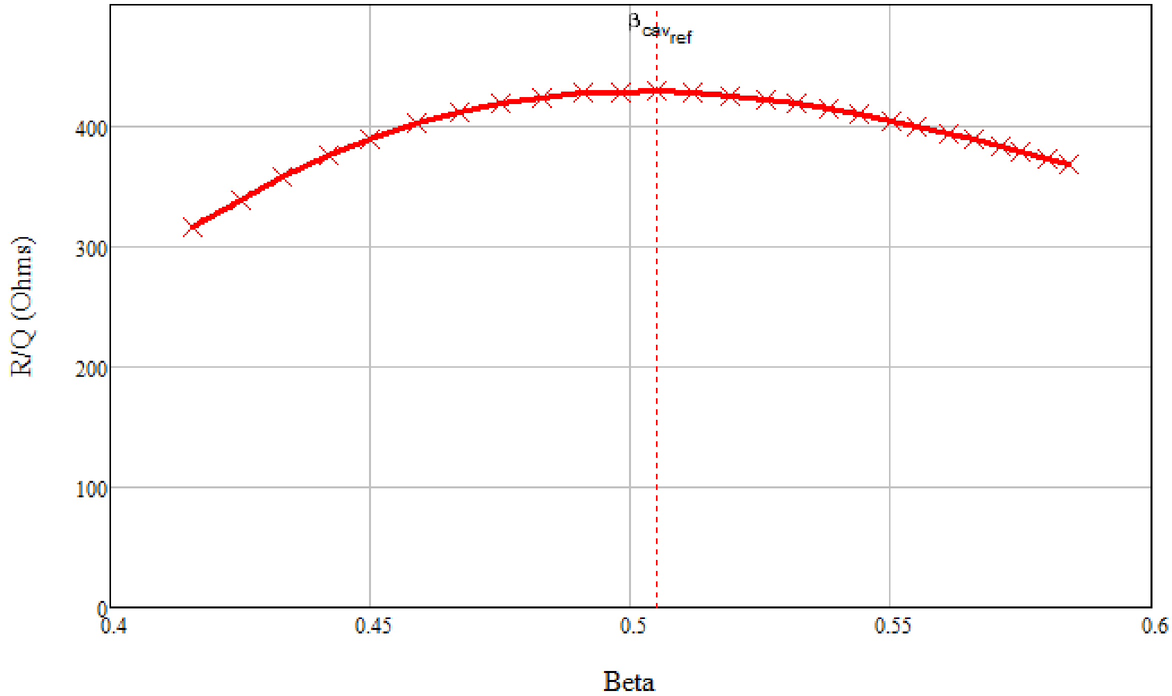

Figure 6. R/Q plot for the Spoke Cavities. The maximum acceleration R/Q of 428 Ohms occurs at a β of 0.505

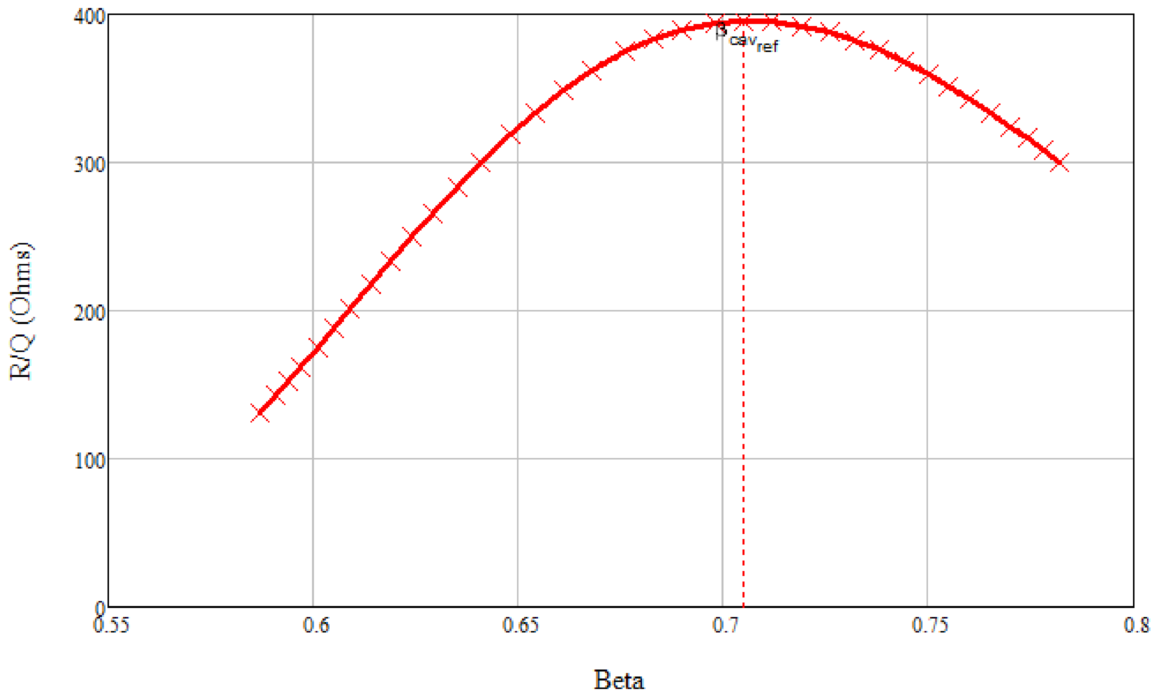

Figure 7. R/Q plot for the Medium Beta Cavities. The maximum acceleration R/Q of 395 Ohms occurs at a β of 0.705

Forward and Reflected Power

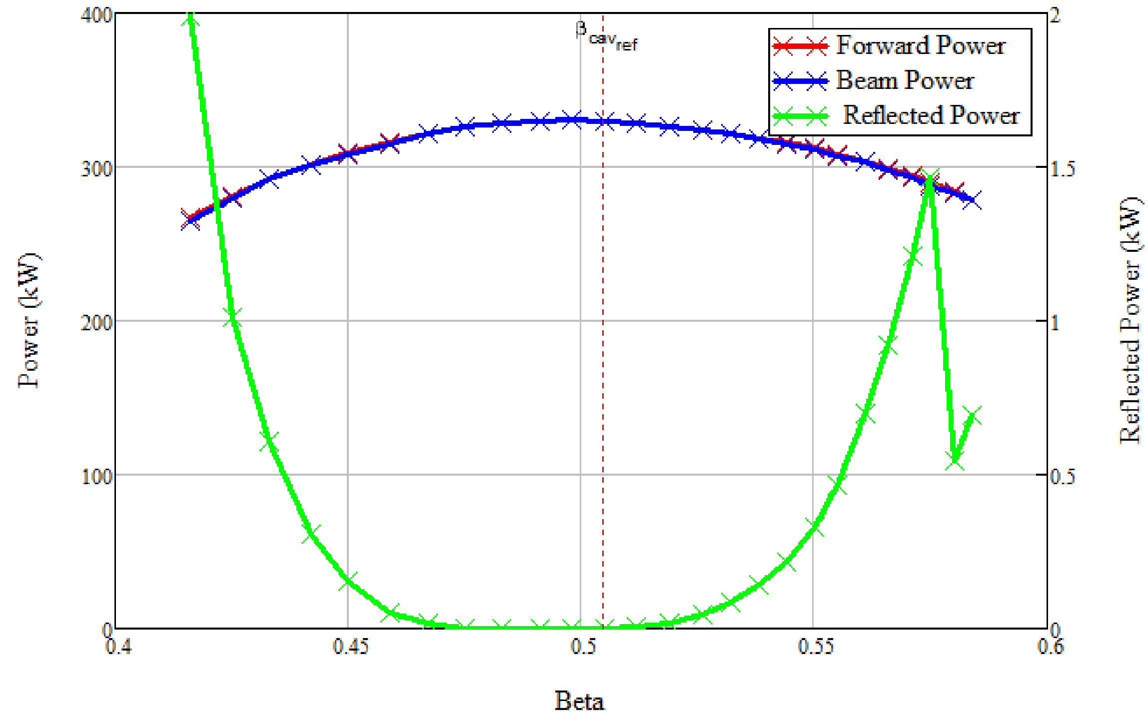

Figure 8. Forward and reflected power for the spoke cavities with a Q_L of $233x10^3$. The Q_L was chosen for the operating point of β=0.505 and a beam current of 62.5 mA.

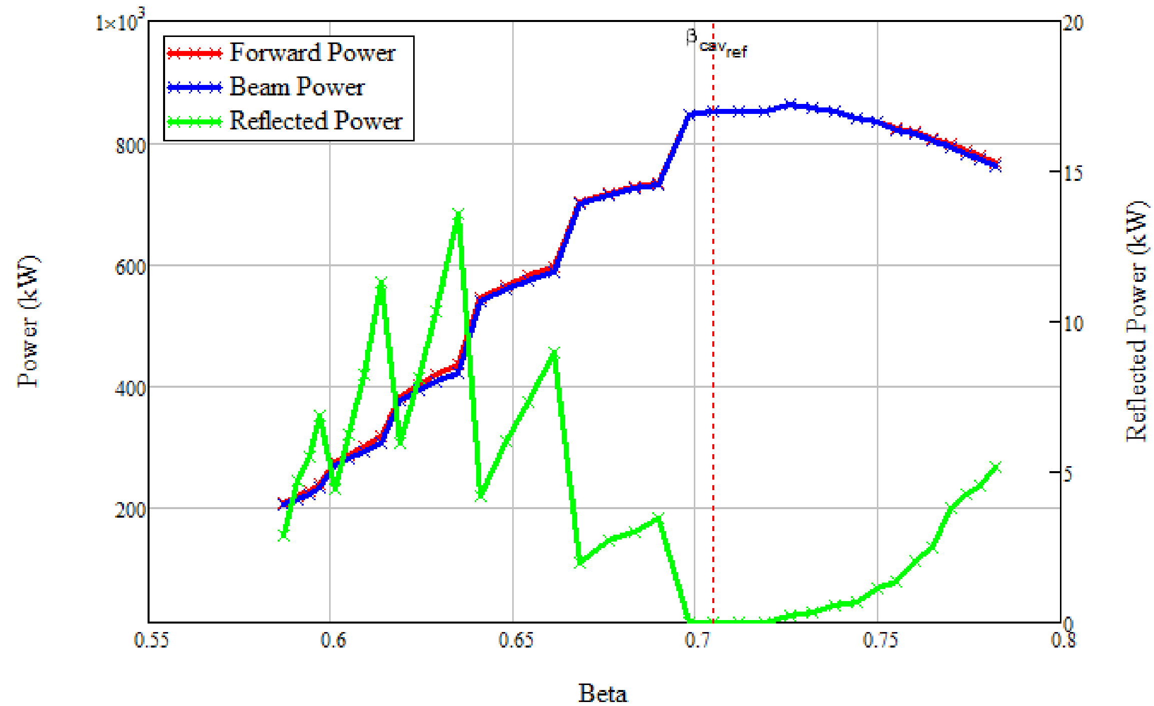

Figure 9. Forward and reflected power for the Medium Beta cavities with a Q_L of $590x10^3$. The Q_L was chosen for the operating point of β=0.705 and a beam current of 62.5 mA.

De-tuning Angle and Synchronous Phase

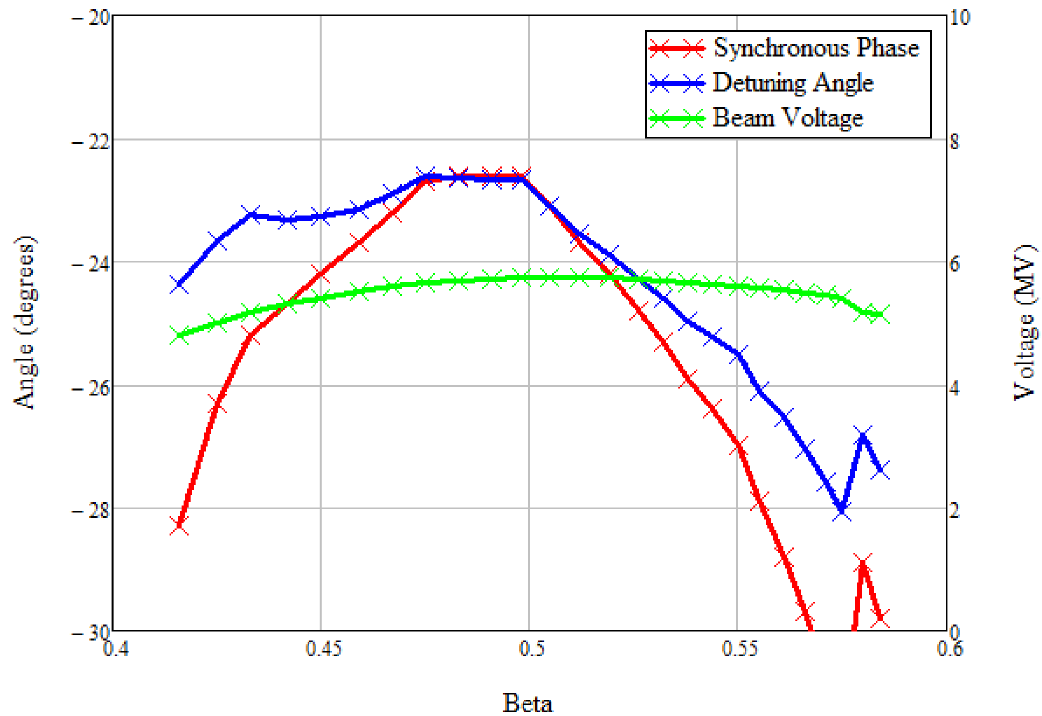

Figure 10. Required detuning angle the Spoke cavities for the operating point of β=0.505. The beam current is 62.5 mA

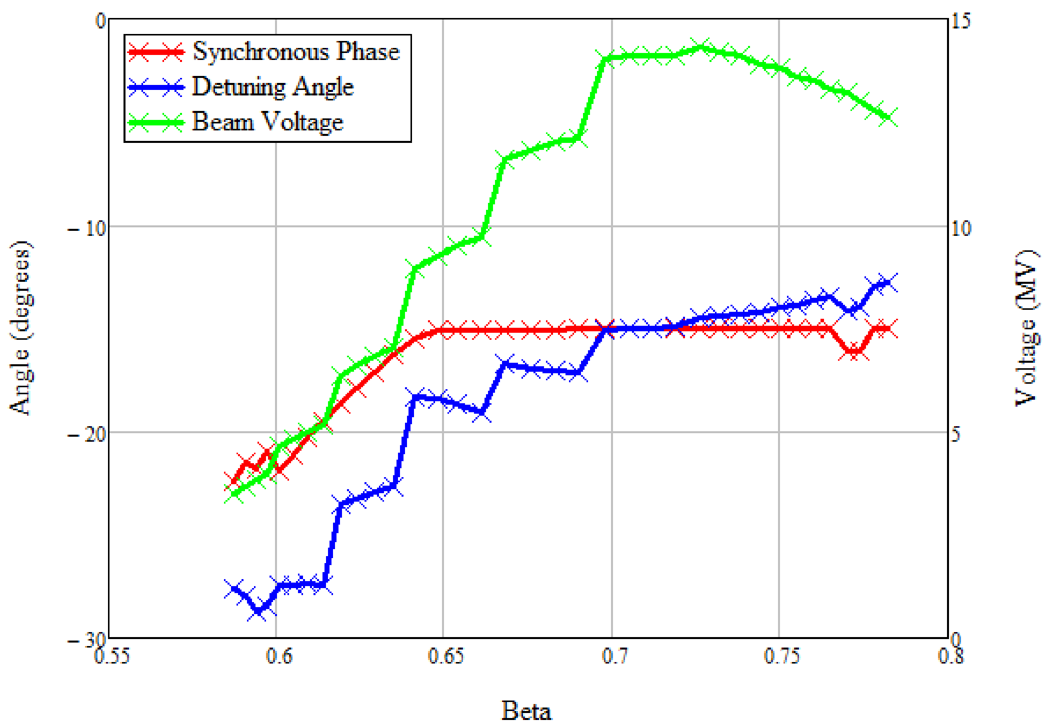

Figure 11. Required detuning angle the Medium Beta cavities for the operating point of β=0.705. The beam current is 62.5 mA