Transit Time Equivalent Circuit for a Superconducting RF Cavity

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Introduction

In most normal conducting accelerating structures, the cell length is tailored for the particle velocity as the beam accelerates through the structure. However, because of the cost and difficulty in fabrication, superconducting structures are grouped into families in which each family is designed for a single beam velocity and then operated over a range of beam velocities. To calculate the optimum loaded Q for a superconducting cavity family, the transit time effects of the beam though the structure must be taken into account. This note will propose an equivalent RF circuit that accounts for transit time effects directly in the circuit.

Generator Equivalent Circuit

An equivalent circuit for the cavity driven only by an RF generator is shown in Figure 1. The coupler is represented as a 1:n transformer which transforms the low impedance and voltage of the generator to a higher impedance and voltage as seen by the cavity. The voltage definition of the cavity can be arbitrary. Here the voltage is defined for a particle transit time T_o. It is important to define this voltage at only one specific transit time because the cavity coupling and generator impedance do not change as a function of particle transit time.

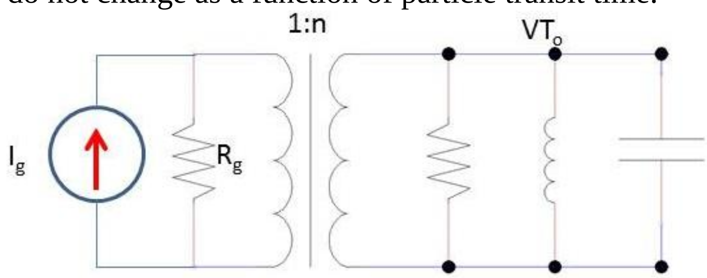

Figure 1. Equivalent Circuit for an RF Cavity driven by a generator

However, the beam interacts differently with the cavity for different transit times through the cavity. This note will propose that the transit time effects are best handled as a transformer between the beam and the cavity as shown in Figure 2. In this way, the generator and cavity losses which do not change as a function of transit time are clearly preserved.

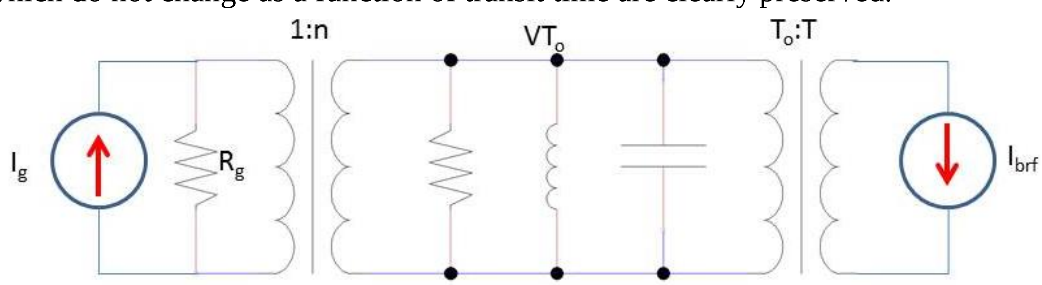

Figure 2. Equivalent Circuit for an RF Cavity driven by a generator and the beam.

The accelerating voltage as seen by the beam is transformed by the transformer ratio $T/T_{o:}$

$$V_{acc}|_{beam} = VT_o \frac{T}{T_o} = VT \quad (1)$$

The circuit from the cavity view point is shown in Figure 3.

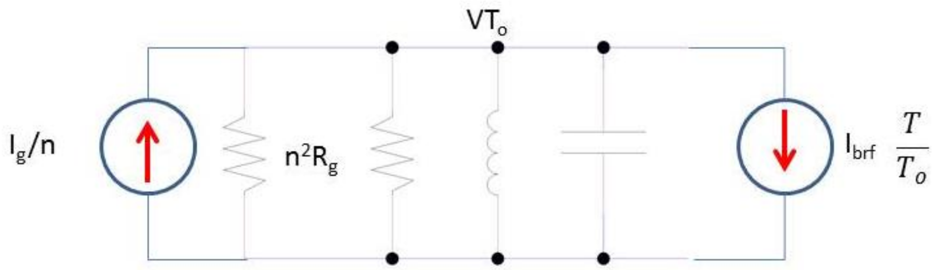

Figure 3. Equivalent Circuit for an RF Cavity driven by a generator and the beam from the cavity point of view.

The cavity voltage can also be separated into fast varying and slow varying parts:

$$VT_o(t) = Re\{\widetilde{VT_o}(t)e^{j\omega t}\} \quad (2)$$

Where the "~" designates a complex phasor. The envelope equation for the complex voltage phasor is given as:

$$\frac{2Q}{\omega_o}\frac{d\widetilde{VT_o}}{dt} + \left(1 - jtan(\varphi_D)\right)\widetilde{VT_o} = n^2R_g\left(\frac{\widetilde{I_g}}{n} - \frac{T}{T_o}\widetilde{I_{brf}}\right) \quad (3)$$

where it has been assumed for superconducting cavities that the shunt resistance of the cavity is much greater than the transformed generator impedance. The static version of Equation 3 is:

$$\left(1 - jtan(\varphi_D)\right)\widetilde{VT_o} = n^2R_g\left(\frac{\widetilde{I_g}}{n} - \frac{T}{T_o}\widetilde{I_{brf}}\right) \quad (4)$$

This note will define the reference of the complex phasors with respect to the cavity voltage:

$$\widetilde{VT_o} = VT_o + 0j \quad (5)$$

$$\widetilde{I_g} = I_g e^{j\phi_g} \quad (6)$$

$$\widetilde{I_{brf}} = I_{brf} e^{j\phi_b} \quad (7)$$

For short bunches

$$I_{brf} = 2I_b \quad (8)$$

where I_b is the beam current.

Cavity Detuning

To minimize the power required from the generator, the generator current and the cavity voltage should be in phase.

$$\phi_g = 0 \tag{9}$$

From Equation 4, setting the generator phase to zero requires that the cavity be detuned to balance off the imaginary part of the beam current:

$$tan(\varphi_D) = \frac{n^2 R_g I_{brf}}{VT}\left(\frac{T}{T_o}\right)^2 sin(\phi_b) \tag{10}$$

Optimum Coupling

From the real part of Equation 4:

$$VT_o = nR_g I_g - n^2 R_g I_{brf}\frac{T}{T_o}cos(\phi_b) \tag{11}$$

For optimum coupling, we would like to choose the coupling factor to minimize the cavity voltage while holding the generator current constant:

$$\frac{dVT_o}{dn} = 0 \tag{12}$$

Also, we should pick the transit time factor $T=T_o$ for the operating conditions to evaluate optimum coupling. The generator current at optimum coupling becomes

$$I_{g_o} = 2nI_{brf_o}cos(\phi_{b_o}) \tag{13}$$

The optimum generator impedance is:

$$n^2 R_g = \frac{V_o T_o}{I_{brf_o}cos(\phi_{b_o})} \tag{14}$$

where the subscript "o" designates a single operating point. Since the generator impedance and coupling factor are fixed at the time of manufacture and cannot be varied during operations, the generator impedance and coupling can only be defined for one cavity voltage, beam current, transit time factor, and synchronous phase angle. For other operating parameters, the cavity will not be operated at optimum coupling.

Calculation of Loaded Q

For a given reference voltage $V_r T_o$, the power dissipated in the circuit is:

$$P_D = \frac{1}{2}\frac{(V_r T_o)^2}{n^2 R_g} \tag{15}$$

where it has been assumed for superconducting cavities that the shunt resistance of the cavity is much greater than the transformed generator impedance. The circuit quality factor is defined as the ratio of the energy stored in the circuit to the amount of energy lost in one cycle.

$$Q = \frac{\omega_r U_r}{P_D} \tag{16}$$

where ω_r is the resonant frequency of the cavity and U_r is the stored energy in the cavity for the voltage $V_r T_o$. The cavity designers provide the data on $V_r T_o$, ω_r, and U_r. From this information, the R/Q for transit time T_o is calculated from Equations 15-16.

$$\left.\frac{n^2 R_g}{Q}\right|_o = \frac{1}{2}\frac{(V_r T_o)^2}{\omega_r U_r} \tag{17}$$

Note that this R/Q is the "circuit" R/Q which is ½ of what is commonly called the "accelerating" R/Q. Using Equation 14 and Equation 17, the loaded Q is:

$$Q_L = \frac{V_o T_o}{\left.\frac{n^2 R_g}{Q}\right|_o I_{brf\,o} cos\left(\phi_{b\,o}\right)} \tag{18}$$

Forward and Reflected Power at Non-Optimum Coupling

At other operating conditions and using Equation 11, the generator current required is

$$I_g = \frac{1}{T/T_o}\frac{VT}{Q_L \left.\frac{n^2 R_g}{Q}\right|_o} + \frac{T}{T_o} I_{brf}\ cos(\phi_b) \tag{19}$$

The forward power from the generator is:

$$P_f = \frac{1}{2} Q_L \left.\frac{n^2 R_g}{Q}\right|_o \left(\frac{I_g}{2}\right)^2 \tag{20}$$

The power delivered to the beam is:

$$P_b = \frac{1}{2} V T I_{brf} cos(\phi_b) \tag{21}$$

The reflected power is calculated from energy conservation:

$$P_r = P_f - P_b \tag{22}$$

The cavity detuning angle is:

$$tan(\varphi_D) = Q_L \left.\frac{n^2 R_g}{Q}\right|_o \frac{1}{VT}\left(\frac{T}{T_o}\right)^2 sin(\phi_b) \tag{23}$$