Power Overhead Calculation for Lorentz Force Detuning

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Abstract

Some measurement at SNS and other labs indicates there might be a large Lorentz force detuning for superconducting cavity, and it gets worse in pulse operation if the repetition rate closes to the mechanical modes. Some data found from other labs about Lorentz force detuning coefficients for different cavities are listed in this note. Some calculations for overhead for Lorentz force detuning in different case are done in this note as well.

It is found that power overhead for detuning within half cavity bandwidth is relatively small under optimal pre-detuning, while overhead rises up sharply if detuning is over one cavity bandwidth. For the spoke cavities at the beginning of the superconducting linac section with low beam velocity, the overhead is much higher than others due to the factor the extra power for Lorentz force detuning is independent on beam velocity. It is also shown that the power overhead for Lorentz force detuning does not change much with Q_L variations.

Introduction

The superconducting cavity operating in high accelerating fields suffers from the LFD (Lorentz force detuning) which results from the interaction of the RF field with the RF wall current. The radiation pressure causes a small deformation of the cavity and consequently leads to a shift of cavity resonance frequency. To compensate this LFD effect, either extra power is required or a piezo tuner is applied.

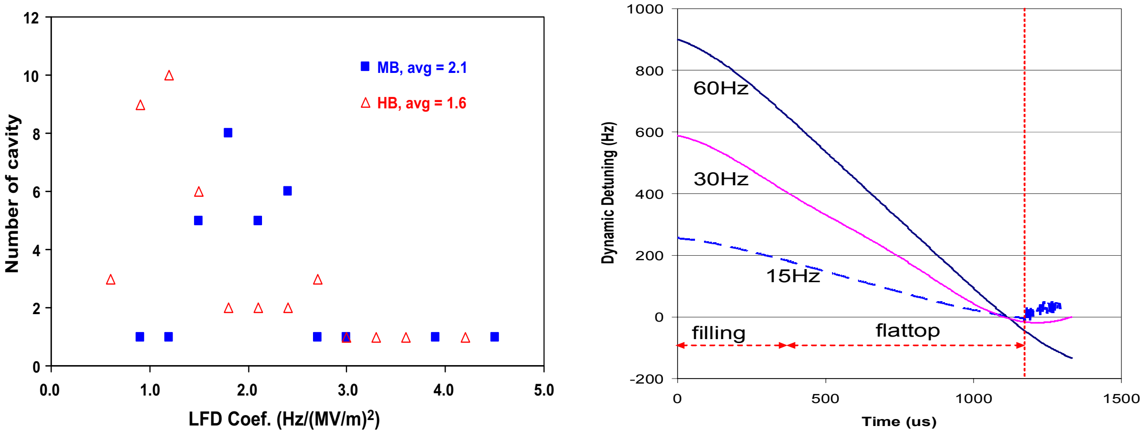
Typical Lorentz Detuning Coefficient

The resonance frequency offset in a cavity due to LFD in steady state is proportional to the square of the cavity field:

$$\Delta f = -K \cdot E_{acc}^2 \tag{1}$$

where K is the Lorentz force detuning coefficient, and E_{acc} is the accelerating field. In general, K is of a order of a few $Hz/\left(MV/m\right)^2$, and the value varies in different cavity design, depending mostly on the wall thickness, material, stiffness structure of the cavity.

At DESY, the detuning coefficient of TESLA-shape 9-cell elliptical cavity with stiffening rings is around 1 $Hz/\left(MV/m\right)^2$ at 35 MV/m pulsed operation[1]. At SNS, detuning coefficients are about $3 \sim 4$ $Hz/\left(MV/m\right)^2$ in medium beta 6-cell cavities and $1 \sim 2$ $Hz/\left(MV/m\right)^2$ in high



(a) Measured dynamic LFD coefficient for all the available SNS superconducting cavities at 2.1 K, with an average acceleration gradient approximately 14 MV/m[4]

(b) Lorentz force detuning at various repetition rates observed in some of the SNS high beta cavities. In this example the accelerating gradient is 12.7 MV/m[2]

Figure 1: LFD measuremetn at SNS

beta 6-cell cavities[2]. In one measurement data at SNS as shown in the left of Figure 1, there are still some individual high beta cavities behaving abnormally with very high LFD coefficient.

Spoke cavities also have significant Lorentz force detuning, but unlike the elliptical cavities whose LFD coefficient is independent of the number of cells, LFD in spoke cavities goes up as the number of loading elements increases[3]. The measured LFD coefficient of a singal spoke cavity ($\beta = 0.21$) at Fermilab is $\sim 4\ Hz/\left(MV/m\right)^2$[5], while the measurement for a triple spoke cavity ($\beta = 0.5$) at ANL is 7.3 $Hz/\left(MV/m\right)^2$[6].

In pulsed mode operation, the frequecy shift of a caivty due to LFD behaviours dynamically, which can be described by the second-order differential equation with mechnical modes:

$$\Delta\ddot{\omega}_n\left(t\right) + \frac{2}{\tau_{m,n}}\Delta\dot{\omega}_n\left(t\right) + \Omega_n^2\Delta\omega_n\left(t\right) = -2\pi K_n\Omega_n^2 \cdot E_{acc}^2\left(t\right) \tag{2}$$

where, Ω_n is the frequency and $\tau_{m,n}$ is the decay constant time of mechanical mode n, K_n is the Lorentz force detuning constant for that mode, $K = \sum_n K_n$, and $\Delta\omega_n$ is the frequency shift caused by mechanical mode n, with the total frequency shift $\Delta\omega = \sum_n \Delta\omega_n$.

If the RF pulse repetition rate, its high-magnitude harmonics or external mechanical vibration frequencies close to the frequencies of mechanical modes, badly cavity oscillation might be caused. Some data measured at SNS as shown in the right of Figure 1 indicates a strong dynamic resonance in some cavities due to high repetition rates operation. Cavity oscillations are also visibly observed at some experiments on FLASH cavities as shown in Figure 2, by using a spare piezo stack in tuner as mechanical sensor.

Power overhead estimation for cavity with optimal Q_L

When the Q_L is optimized for the cavity and appropriate pre-detuning is chosen to completely cancel the synchronous phase effect, the power needed for the cavity to maintain a desired

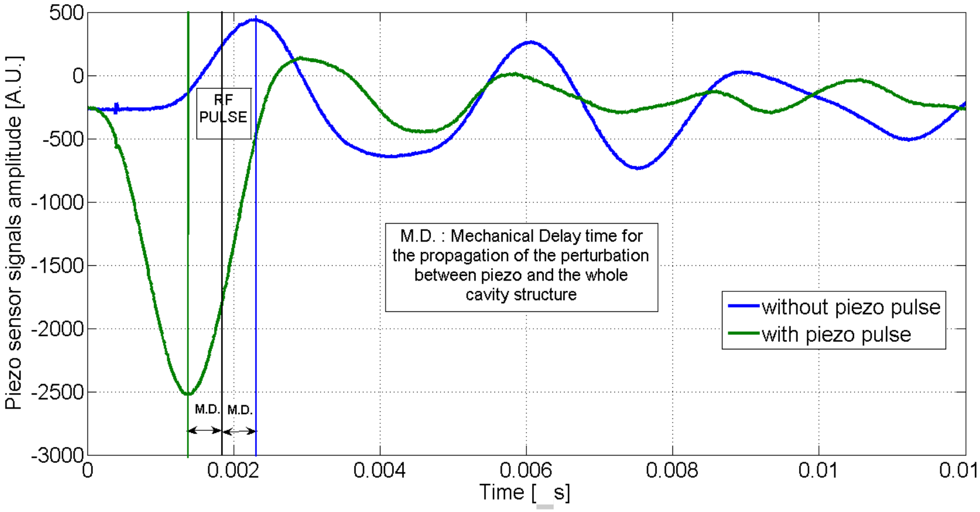


Figure 2: Measurement of cavity mechanical oscillation at FLASH with/without peizo tuner compensation, cavity operating at gradient 35 MV/m[7]

accelerating field can be calculated as follows[8]:

$$P_g = \frac{1}{8}\frac{V_{cav}^2}{R_L}\left(4 + \left(\frac{\Delta\omega_L\left(t\right)}{\omega_{1/2}}\right)^2\right), \tag{3}$$

where $\Delta\omega_L\left(t\right)$ is the dynamic cavity resonance frequency offset due to LFD. At ESS with very long beam pulse of around 3 ms, the dynamic frequency offset can be roughly considered as that it changes from 0 to $K \cdot E_{acc}^2$ assuming there is no badly mechanical oscillation with the cavity. If we define the power overhead with the ratio of the maximum extra power compensating the LFD to the required generator power in the case without LFD, then the power overhead can be estimated as $\left(\frac{K \cdot E_{acc}^2}{f_{1/2}}\right)^2$. The overhead can be reduced by a factor of 4 if we manage to adjust the pre-detuning for LFD right into the middle of the dynamic frequency offset range during beam pulse. Table 1, 2, and 3 list respectively the estimated power overhead for high beta, low beta and spoke cavities under different LFD coefficient K, in which both cases with and without optimal pre-detuning for LFD are given (Δf is the frequency offset due to LFD, φ_D the detuning angle, and $f_{1/2}$ the cavity half bandwidth with $Q_L = 7.878 \times 10^5$ for high beta, $Q_L = 7.973 \times 10^5$ for med beta, and $Q_L = 1.5 \times 10^5$ for spoke).

It is found that the power overhead depends strongly on the factor $\Delta f / f_{1/2}$, and the overhead is relatively small if the detuning is below half cavity bandwidth ($\Delta f < f_{1/2}$) together with optimal pre-detuning for LFD, while the overhead goes very high if the detuning is over one cavity bandwidth ($\Delta f > 2f_{1/2}$), more than 25% under optimal pre-detuning and more than 100% under no pre-detuning for LFD. Therefore, in the later sections, $\Delta f = f_{1/2}$ (correspondingly K $\sim$ 1.5 for high beta and K $\sim$ 2 for low beta cavity) and $\Delta f = 2f_{1/2}$ (correspondingly K $\sim$ 3 for high beta and K $\sim$ 4 for low beta cavity) are taken respectively as different degree of LFDs to estimate the power overheads needed. For the spoke cavity of high bandwidth and a low accelerating field the effect of LFD is not as significant as elliptical cavity, in which the detuning does not reach the half bandwidth $f_{1/2}$ until LFD coefficient K $\sim$ 17 $Hz/(MV/m)^2$.

Table 1: Overhead estimation under different K for high beta cavity ($E_{acc} = 18 MV/m$)

K ($Hz/(MV/m)^2$)	Δf (Hz)	$f_{1/2}$ (Hz)	$\Delta f/f_{1/2}$	φ_D (°)	Overhead w.o. predetuning	Overhead with predetuning
1	324	447	0.7	35.9	13.1%	3.3%
1.5	486	447	1.1	47.4	29.5%	7.4%
2	648	447	1.4	55.4	52.5%	13.1%
2.5	810	447	1.8	61.1	82.1%	20.5%
3	972	447	2.2	65.3	118.2%	29.5%
3.5	1134	447	2.5	68.5	160.8%	40.2%
4	1296	447	2.9	71.0	210.1%	52.5%

Table 2: Overhead estimation under different K for med beta cavity ($E_{acc} = 15 MV/m$)

K ($Hz/(MV/m)^2$)	Δf (Hz)	$f_{1/2}$ (Hz)	$\Delta f/f_{1/2}$	φ_D (°)	Overhead w.o. predetuning	Overhead with predetuning
1	225.0	442	0.5	27.0	6.5%	1.6%
1.5	337.5	442	0.8	37.4	14.6%	3.6%
2	450.0	442	1.0	45.5	25.9%	6.5%
2.5	562.5	442	1.3	51.9	40.5%	10.1%
3	675.0	442	1.5	56.8	58.4%	14.6%
3.5	787.5	442	1.8	60.7	79.4%	19.9%
4	900.0	442	2.0	63.9	103.8%	25.9%

Table 3: Overhead estimation under different K for spoke cavity ($E_{acc} = 8.5 MV/m$)

K ($Hz/(MV/m)^2$)	Δf (Hz)	$f_{1/2}$ (Hz)	$\Delta f/f_{1/2}$	φ_D (°)	Overhead w.o. predetuning	Overhead with predetuning
1	72.25	1174	0.06	3.5	0.09%	0.02%
5	361.25	1174	0.31	17.1	2.37%	0.59%
9	650.25	1174	0.55	29.0	7.67%	1.92%
13	939.25	1174	0.80	38.7	16.00%	4.00%
17	1228.25	1174	1.05	46.3	27.36%	6.84%

Power overhead calculation for non-optimal Q_L

For the proton linac, Q_L is not optimal for each cavity but usually set to a fixed design value for the same type of cavities. In the case of non-optimal Q_L and non-optimal beam velocity for the cavity, the required generator power P_g and power overhead ρ_{max} for LFD can be calculated as follows (assuming that proper pre-detuning has been applied to cancel the effect of synchronous phase operation)[8]:

$$P_g = \frac{1}{8}\frac{V_{cav}^2}{R_L}\left\{\left(1+\frac{R_L}{V_{cav}}\left(\beta\right)I_b cos\varphi_b\right)^2+\left(\frac{\Delta\omega_L\left(t\right)}{\omega_{1/2}}\right)^2\right\} \tag{4}$$

$$\rho_{max}\left(\beta\right) = \left(\frac{\left|\Delta\omega_L\left(t\right)\right|_{max}}{\omega_{1/2}}\cdot\frac{1}{1+0.5(R/Q)_\beta Q_L I_b cos\varphi_b/V_{cav}}\right)^2 \tag{5}$$

As mentioned in last section, the detuning range $\Delta f = f_{1/2}$ and $\Delta f = 2f_{1/2}$ will be assumed respectively to estimate power overhead for LFDs in different levels. Figure 3 shows the correspoding overhead in half bandwidth detuning for all the superconducting cavities, by employing equation 5 and assuming optimal pre-detuning for LFD is applied. It should be noted that the term $\frac{V_{cav}^2}{R_L}$ is dependent not on beam velocity β but on the accelerating field, which means that for the same detuning factor $\Delta f/f_{1/2}$ in the same type of cavity operating at same accelerating field, the extra power for LFDs are the same. In contrast, the generator power P_g required under no LFD does depend on β and P_g goes up as β increases. As a result, the ratio of the extra power for LFD to generator power without LFD decreases as β increases, which is the case in Figure 3 for spoke and most high beta cavities operating nearly at the same accelerating fields. In the med beta cavities, power overhead trend behaviours differently as they are designed to operate at much different accelerating fields.

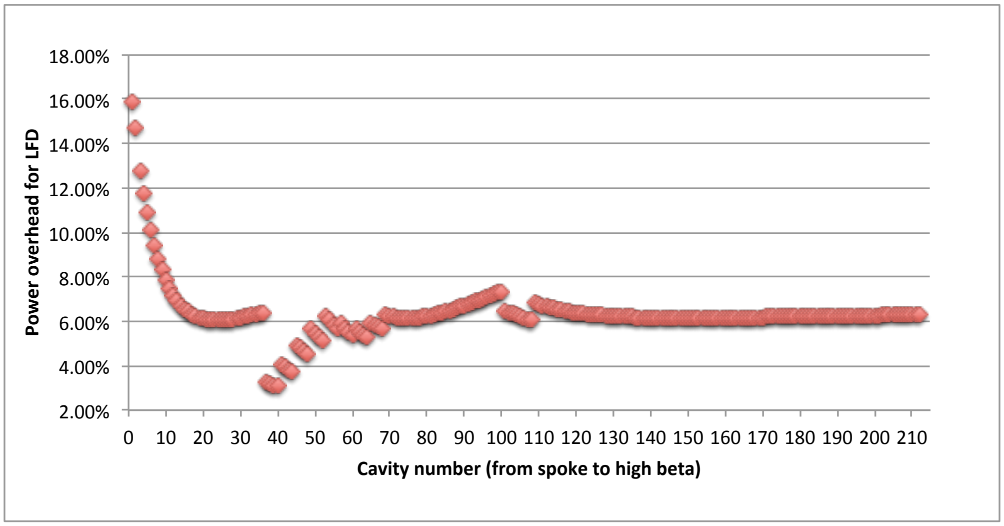


Figure 3: Power overhead required for LFD of half cavity bandwidth ($\Delta f = f_{1/2}$) for all the superconducting cavities including spoke, med beta and high beta

Although in spoke cavity a relatively high overhead is required at the level of half bandwidth detuning, the situation is not getting worse due to its high bandwidth and low accelerating field operation. However, the LFD coefficient is still suggested to keep low enough so as to have the factor $\Delta f/f_{1/2}$ lower than 0.5 (correspondingly K $\sim 8\ Hz/(MV/m)^2$). More attention is paid to elliptical cavity in the later sections due to its potential risk at high LFD. The power overhead required goes up sharply with the square of the factor $\Delta f/f_{1/2}$ as the detuning Δf increases. Figure 4 shows a comparison of power overheads required for LFDs of half cavity bandwidth ($\Delta f = f_{1/2}$) and of one bandwidth ($\Delta f = 2f_{1/2}$), in both of which optimal pre-detunings for LFD are assumed.

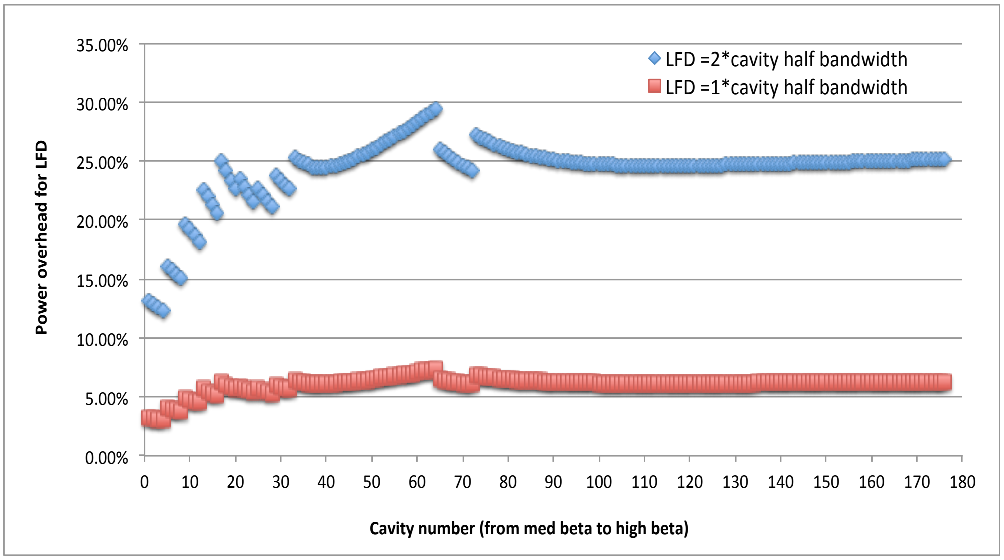


Figure 4: Comparision of power overheads required for LFDs of half cavity bandwidth ($\Delta f = f_{1/2}$) and of one bandwidth ($\Delta f = 2f_{1/2}$), for med beta and high beta cavities

Power overhead calculation for Q_L variations

In the case of Q_L varies from designed value in practice due to some uncertainties, the generator power and power overhead for LFD will change correspondingly. Figure 5 and 6 show the power overhead required in Q_L variations at half bandwidth and one bandwidth detuning separately, in which fixed detuning Δf is assumed for different Q_L variations.

It can be seen from Figures that the power overhead does not change so much as Q_L varies in $\pm 5\%$. The overhead here defined is still the ratio of extra power for LFD to the generator power needed without LFD. A major contribution to the LFD overhead change is that the cavity half bandwidth $f_{1/2}$ is different under different Q_L, resulting in the change of factor $\Delta f/f_{1/2}$. The extra power due to Q_L variations is not separated in LFD overhead calculation from the total generator power needed without LFD, which decreases if Q_L goes close to the $Q_{L,opt}$, while increases if Q_L goes far away from the $Q_{L,opt}$.

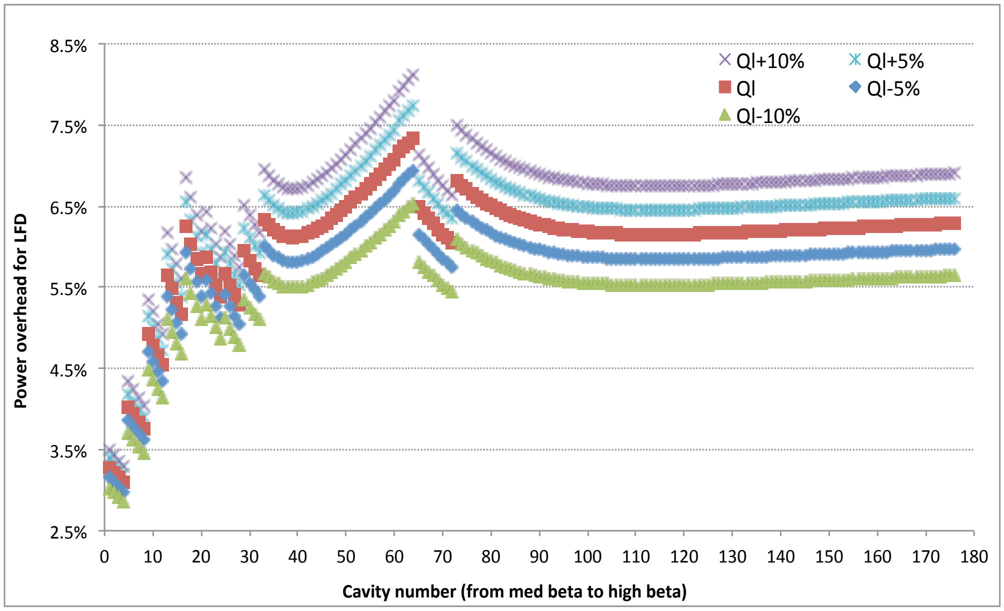


Figure 5: Power overhead required in Q_L variations for LFD of half cavity bandwidth ($\Delta f = f_{1/2}$ under designed Q_L, and is kept the same for different variations)

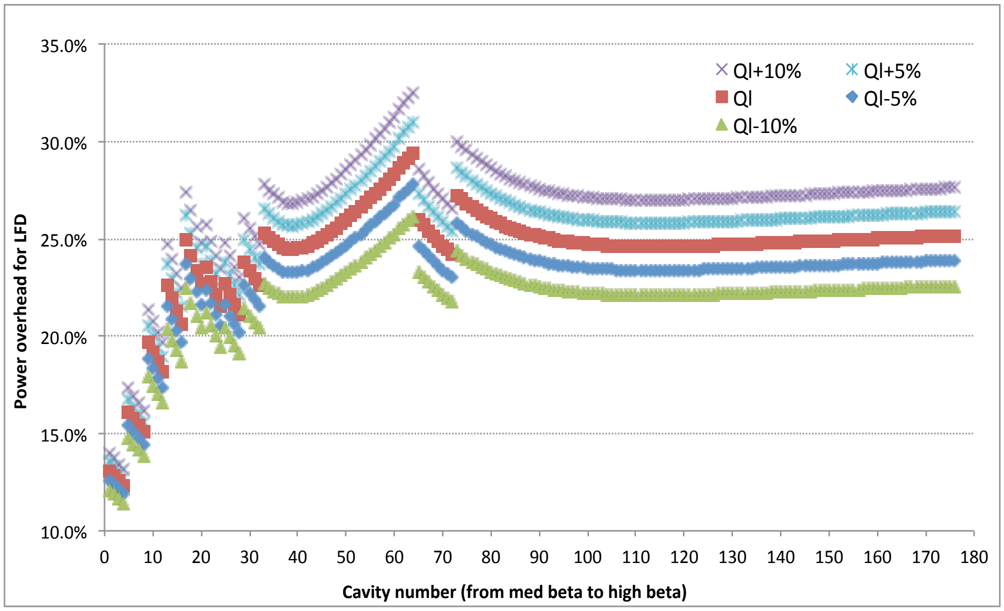


Figure 6: Power overhead required in Q_L variations for LFD of one cavity bandwidth ($\Delta f = 2f_{1/2}$ under designed Q_L, and is kept the same for different variations)

Conclusion

Some measurements at SNS show that the detuning coefficients for elliptical cavity are up to 4 $Hz/\left(MV/m\right)^2$, and it gets worse when the repetition rate goes up to 60 Hz in some cavities due to mechanical resonance. Measurements on spoke cavities at Fermilab and ANL show the LFD coefficient is around $4 \sim 8Hz/\left(MV/m\right)^2$.

Some calculation indicates in this note that power overhead for LFD rises sharply when the detuning is over one cavity bandwidth (detuning angle over 60 °) even with optimal pre-detuning. The overhead could be reduced to a small value if the detuning is limited into half cavity bandwidth by constraining LFD coefficient together with optimal pre-detuning for LFD, and avoiding mechanical resonance as well. Certainly, more overhead will be reduced if piezo tuner is applied and works well to reduce the detuning less than 1/4 cavity bandwidth.

For the spoke cavity section, several cavities at the beginning of section with low beam velocity required more overhead for LFD due to the facor that extra power for compensating LFDs are the same in the same type of cavity operating at same accelerating field but the generator power without LFD is goes down in low beam velocity β. It is therefore helpful to have these cavities of low LFD coefficient.

The power overhead changes for LFD in Q_L variation of ±5% and ±10% are calculated as well, and it is found that there are only around ±1% ∼ ±3% changes in overhead for different levels of LFD.

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