Some Considerations on Predetuning for Superconducting Cavity

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1 Introduction

Some analysis of pre-detuning for different cases for superconducting cavity are done in this note. Only feedforward is applied in simulation to try to find appropriate beam injection time and required steady state generator current. For synchronous phase effect and Q_L spread, an optimized pre-detuning is helpful to minimize the generator power. It is also examined that resonance tracking at filling stage is an effective way to achieve required steady state while not consuming extra generator power or increasing filling time. For Lorentz force detuning, choosing an appropriate pre-detuning is essential, by which extra power could be reduced by almost a factor of 4.

2 Predetuning for synchronous phase

In a RLC cavity model, at steady state:

$$\mathbf{V_{cav}} = \frac{R_L}{1 - itan\varphi_D} \cdot \mathbf{I_{total}} \tag{1}$$

where, $R_L = \frac{1}{2}\left(R/Q\right)Q_L$, $tan\varphi_D = Q_L\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)$, $\mathbf{I_{total}} = \mathbf{I_g} - \mathbf{I_b}$, $\mathbf{I_g} = 2\mathbf{I_{for}}$, $\mathbf{I_b} = 2\mathbf{I_{b0}}$. $\mathbf{I_{for}}$ is equivalent klystron forward current seen from cavity side while $\mathbf{I_{b0}}$ is the average DC beam current.

If we take the cavity voltage as the reference, i.e.,$\mathbf{V_{cav}} = V_{cav} + i \cdot 0$, and correspondingly write the current with the complex form:

$$\begin{aligned} \mathbf{I_g} &= I_{gr} + iI_{gi} \\ \mathbf{I_b} &= I_{br} + iI_{bi} \end{aligned} \tag{2}$$

giving the known V_{cav} and the detuning of the cavity, we can conclude the required **Ig** as follows:

$$I_{gr} = \frac{V_{cav}}{R_L} + I_b cos\varphi_b \tag{3}$$

$$I_{gi} = -\frac{V_{cav}}{R_L} tan\varphi_D - I_b sin\varphi_b \tag{4}$$

Minimum power is required when $I_{gi} = 0$ and $I_g^2 R_L$ is minimized by optimizing Q_L, i.e.,

$$Q_{L,opt} = \frac{V_{cav}}{0.5\,(R/Q)\,I_b cos\varphi_b} \tag{5}$$

$$tan\varphi_D = -tan\varphi_b \tag{6}$$

where, φ_b is the synchronous phase.

Ideally, the steady state is achieved immediately when having the beam injected in a proper time $t_{inj} = \tau ln2$.

3 Predetuning during filling stage

At filling stage, the cavity is behaving as transient intermediate state as the cavity voltage rises gradually to the nominal value. During the filling time, the pre-detuning keeps working but there is no beam compensating this, which leads to a perturbation to steady state as shown in the left figure of Figure 1. To avoid this perturbation, one effective way is to track the cavity resonant frequency by modulating the phase of the klystron input signal as $\varphi = \int \Delta\omega \; dt$. In this case, the cavity amplitude response identically to the resonant case, but the cavity phase is increasing by the term $\int \Delta\omega \; dt$. It has to be noted that applying a proper phase offset is important to make the phase approaching to the design value at beam injection time.

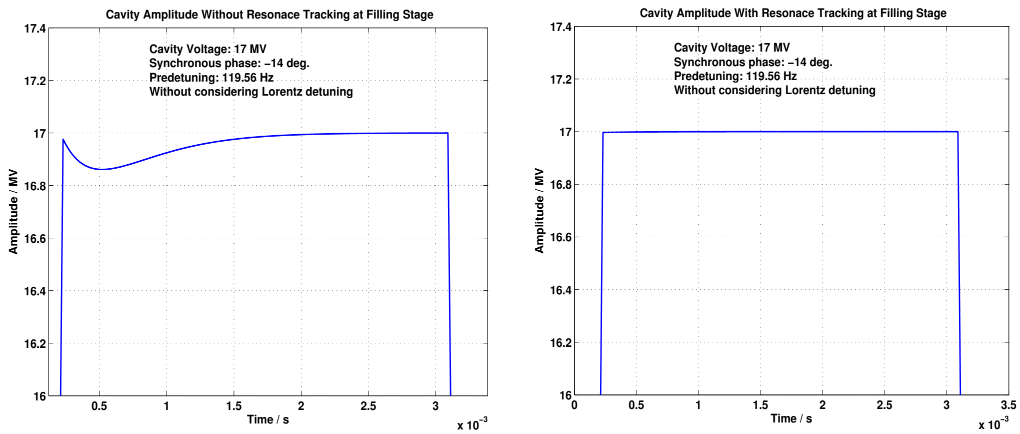


Figure 1: Cavity amplitude without/with resonance tracking at filling stage

4 Predetuning for Q_L spread

Some extra power is required when the Q_L is not optimized but fixed to a reference value. Minimum required power is reached with $I_{gi} = 0$ in equation 4 via proper pre-detuning:

$$tan\varphi_D = -\frac{R_L I_b sin\varphi_b}{V_{cav}} = -\frac{Q_L}{Q_{L,opt}} tan\varphi_b \tag{7}$$

for the superconducting cavity, $tan\varphi_D \approx 2Q_L \frac{\Delta\omega}{\omega_{RF}}$ and $R_L = \frac{1}{2}\left(R/Q\right)Q_L$. To let $I_{gi} = 0$, then we have,

$$\Delta\omega = -\frac{\omega_{RF}\left(R/Q\right)I_b sin\varphi_b}{4V_{cav}} \tag{8}$$

it can be seen clearly that equation 8 is independent with Q_L, which means I_{gi} is kept 0 under constant pre-detuned frequency $\Delta\omega$, no matter how much Q_L varies.

However, Q_L variation indeed affect the beam injection time. In order to reach the voltage required at flat top, the beam injection time is chosen as follows (RF signal is tracking the resonant frequency during filling stage):

$$V_{cav} = I_g R_L \left(1 - e^{-\frac{t_{inj}}{\tau}}\right) \tag{9}$$

where $\tau = \frac{2Q_L}{\omega_0}$. Assuming that the generator current is kept the same at the filling stage and at the injection time, we can get t_{inj} by combining equation

3 and equation 9:

$$t_{inj} = \tau ln\left(\frac{I_{gr}}{I_b cos\varphi}\right) \tag{10}$$

It should be noted that the t_{inj} obtained in equation 10 can not apply in the case without resonance tracking at filling stage, in which it is hard to keep a constant voltage by only adjusting the t_{inj} under constant generator current. A further analysis about injection time has been made in appendix from a viewpoint of transient analysis. The injection time could be changeable at a cost of consuming more generator power or prolonging the filling time. In that case, the phase and amplitude of the voltage at the end of filling must equals that of the steady state voltage in order to keep a constant voltage after beam injection.

5 Predetuning for non-optimal beam velocity

In the case that the cavity length is not optimized for the velocity of the beam passing through the cavity, the beam obtained accelerating voltage V_{cav} is a function of beam velocity factor β:

$$V_{cav}\left(\beta\right) = V_{cav}\left(\beta_0\right) \cdot \frac{4}{5\pi}\frac{\beta^2}{\beta_0^2 - \beta^2} cos\left(\frac{\beta_0}{\beta}\frac{5\pi}{2}\right) \tag{11}$$

where $V_{cav}\left(\beta_0\right)$ is the voltage of the cavity which is optimally designed to match the beam velocity. If we define the variable $T_b\left(\beta\right)$ to replace the long term $\frac{4}{5\pi}\frac{\beta^2}{\beta_0^2-\beta^2} cos\left(\frac{\beta_0}{\beta}\frac{5\pi}{2}\right)$, then equation (11) becomes:

$$V_{cav}\left(\beta\right) = V_{cav}\left(\beta_0\right) T_b\left(\beta\right) \tag{12}$$

Meantime, $R/Q, R_L$ is also varying as the beam velocity changes from its optimal value in different cavities, which can be presented as below:

$$\left(R/Q\right)\left(\beta\right) = \left(R/Q\right)\left(\beta_0\right) T_b^2\left(\beta\right) \tag{13}$$

$$R_L\left(\beta\right) = R_L\left(\beta_0\right) T_b^2\left(\beta\right) \tag{14}$$

Some other items used frequently in this paper are:

$$\frac{V_{cav}}{R_L}\left(\beta\right) = \frac{V_{cav}}{R_L}\left(\beta_0\right) \cdot \frac{1}{T_b\left(\beta\right)} \tag{15}$$

$$\frac{V_{cav}^2}{R_L}\left(\beta\right) = \frac{V_{cav}^2}{R_L}\left(\beta_0\right) \tag{16}$$

For the non-optimal beam velocity, we can still pre-detune the cavity to minimize the generator power (make $I_{gi} = 0$) as done in the last section according to equation 7. The optimal $Q_{L,opt}$ for the cavity not matching the beam velocity could be:

$$Q_{L,opt}\left(\beta\right) = \frac{V_{cav}}{0.5\left(R/Q\right)\left(\beta\right) I_b cos\varphi_b} \tag{17}$$

then, the proper pre-detuning value in this case can be calculated as follows:

$$tan\varphi_D = -\frac{R_L\left(\beta\right) I_b sin\varphi_b}{V_{cav}} = -\frac{Q_L}{Q_{L,opt}\left(\beta\right)} tan\varphi_b \tag{18}$$

6 Predetuning for Lorentz detuning

The dynamic detuning of a cavity due to lorentz force can be described by the first order differential equation:

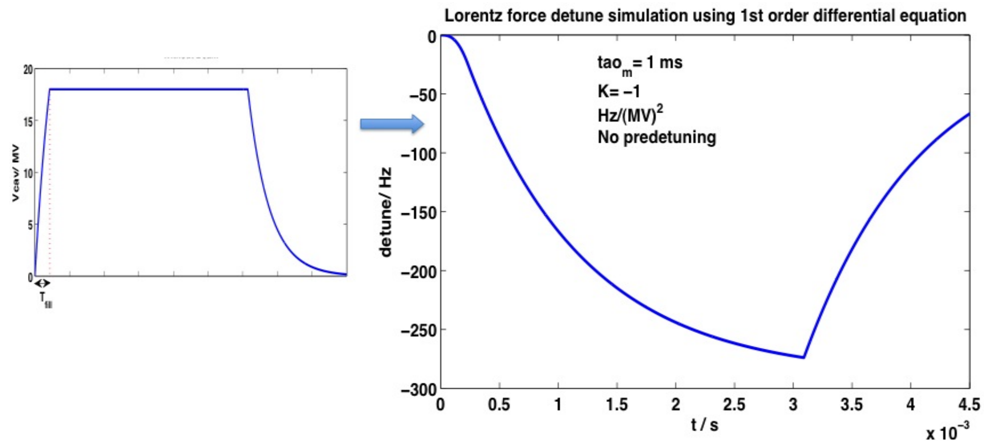
$$\tau_m \Delta\dot{\omega}\left(t\right) + \Delta\omega\left(t\right) = -2\pi K \cdot E_{acc}^2\left(t\right) \tag{19}$$

where, τ_m is the mechanical time constant, K the Lorentz force detuning constant, E_{acc} the accelerating field, and $\Delta\omega\left(t\right) = \omega_0\left(t\right) - \omega_{RF}$. The first order differential equation can well describe the Lorentz force detuning but a more accurate result might be obtained by using the second order differential equation having known the mechanical modes:

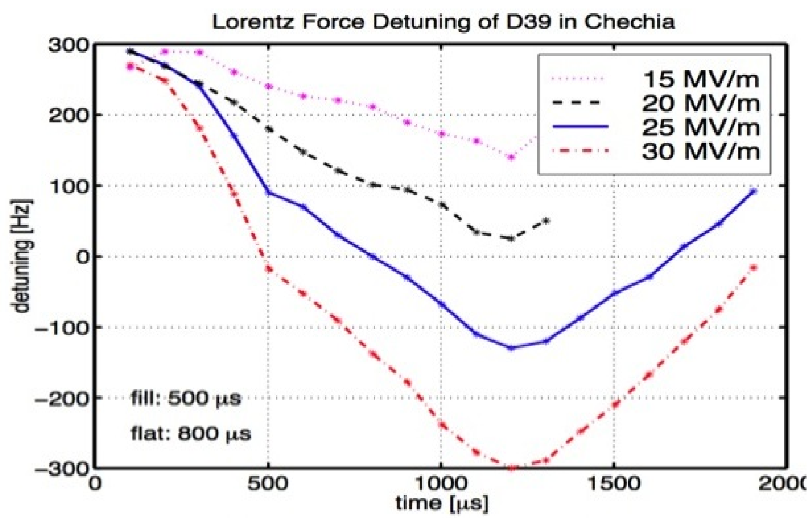
$$\Delta\ddot{\omega}_n\left(t\right) + \frac{2}{\tau_{m,n}}\Delta\dot{\omega}_n\left(t\right) + \Omega_n^2 \Delta\omega_n\left(t\right) = -2\pi K_n \Omega_n^2 \cdot E_{acc}^2\left(t\right) \tag{20}$$

where, Ω_n is the frequency and $\tau_{m,n}$ is the decay constant time of mechanical mode n, K_n is the Lorentz force detuning constant for that mode, $K = \sum_n K_n$, and $\Delta\omega_n$ is the frequency detuning caused by mechanical mode n, with the total frequency detuning $\Delta\omega = \sum_n \Delta\omega_n$.

A simulation using 1st order differential equation for high beta superconductivity cavity is shown in the upper figure of Figure 2, in which the pattern



(a)



(b)

Figure 2: Lorentz detuning simulation(a) and real data in TESLA cavity(b)

of the detuning is very similar with the real data measured in Desy's cavity as listed in the bottom figure of Figure 2.

To compensate the detuning, feedfoward algorithm is employed in simulation based on equation 3 and 4. When Lorentz detuning exists, the imaginary part of the generator current $\mathbf{I}_g$ can be no longer kept zero, which means extra power is required for this kind of detuning if there is no piezo tuner functioning. We here write the generator power $P_g = \frac{1}{2}|\mathbf{I}_{for}|^2 R_L = \frac{1}{8}|\mathbf{I}_g|^2 R_L$ below:

$$\begin{aligned} P_g &= \frac{1}{8}\left(I_{gr}^2 + I_{gi}^2\right) R_L \\ &= \frac{1}{8} R_L \left\{ \left(\frac{V_{cav}}{R_L} + I_b cos\varphi_b \right)^2 + \left(-\frac{V_{cav}}{R_L} tan\varphi_D - I_b sin\varphi_b \right)^2 \right\} \end{aligned} \tag{21}$$

In superconducting cavity, $tan\varphi_D \approx \frac{\Delta\omega}{\omega_{1/2}}$, $\Delta\omega = \Delta\omega_P + \Delta\omega_L(t)$, $\Delta\omega_P$ is the pre-detuning to compensate the synchronous phase operation while $\Delta\omega_L(t)$ is due to Lorentz force detuning. When the Q_L is optimized and appropriate pre-detuning is chosen to completely cancel the synchronous phase effect as that shown in equation 5 and 6, the generator power can be written as:

$$P_g = \frac{1}{8}\frac{V_{cav}^2}{R_L}\left(4 + \left(\frac{\Delta\omega_L(t)}{\omega_{1/2}}\right)^2\right) \tag{22}$$

It is seen from the Figure 2 that $|\Delta\omega_L(t)|$ is increasing all the way and $\Delta\omega_L(t)$ is always negative during the beam pulse. A positive pre-detuning is therefore introduced in superconducting cavity to optimize the total effect of the Lorentz force detuning on cavity. Obviously, a postive pre-detuning for Lorentz force effect with around half of $|\Delta\omega_L(t)|_{max}$ can reduce the peak extra power most, nearly by a factor of 4. If a maximum detuning $|\Delta\omega_L(t)|_{max}$ is as high as the cavity half bandwidth $\omega_{1/2}$, then up to 25% extra power will be required to regulate the cavity field without pre-detuning, while only 6.25% extra power is needed with the right pre-detuning.

Simulations on Lorentz force detuning compensation by feedforward have been done and it is shown clearly in Figure 3 that extra power for the compensation can be well reduced under proper pre-detuning. To make the comparison in detail, the total cavity detuning under different pre-settings are also shown in Figure 4.

In the case of non-optimal Q_L, the generator power is (assuming that proper pre-detuning has been applied to cancel the effect of synchronous phase operation):

$$P_g = \frac{1}{8}\frac{V_{cav}^2}{R_L}\left\{\left(1+\frac{R_L}{V_{cav}}I_b cos\varphi_b\right)^2+\left(\frac{\Delta\omega_L(t)}{\omega_{1/2}}\right)^2\right\} \tag{23}$$

Thus the ratio of extra power for Lorentz force detuning to generator power needed in the case without Lorentz detuning can be calculated as below:

$$\begin{aligned}\rho &= \left(\frac{\Delta\omega_L(t)}{\omega_{1/2}}\cdot\frac{1}{1+R_L I_b cos\varphi_b/V_{cav}}\right)^2 \\ &= \left(\frac{\Delta\omega_L(t)}{\omega_{1/2}}\cdot\frac{1}{1+Q_L/Q_{L,opt}}\right)^2\end{aligned} \tag{24}$$

While for the case of non-optimal beam velocity, the ratio turns to be:

$$\rho(\beta) = \left(\frac{\Delta\omega_L(t)}{\omega_{1/2}}\cdot\frac{1}{1+Q_L/Q_{L,opt}(\beta)}\right)^2 \tag{25}$$

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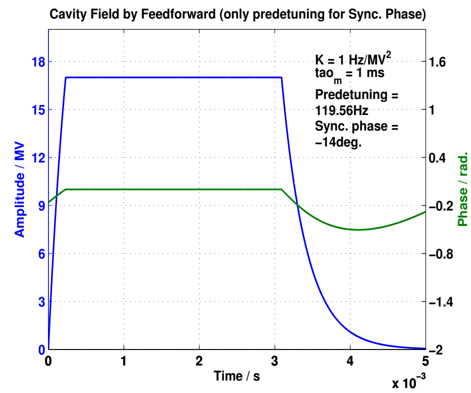
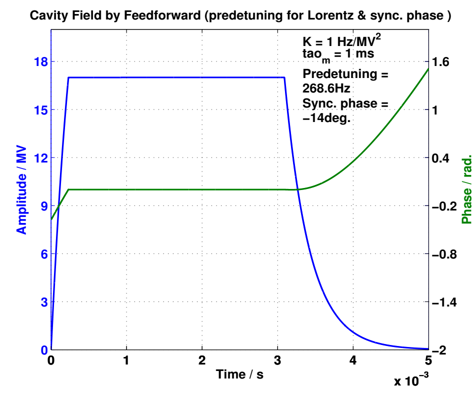


(a)



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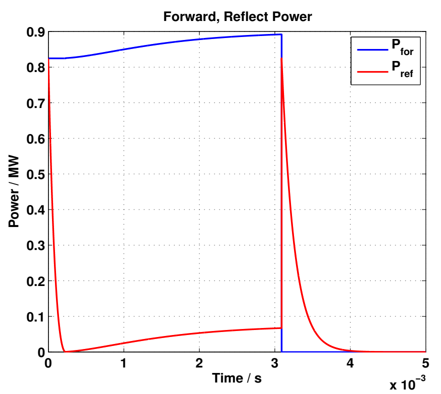

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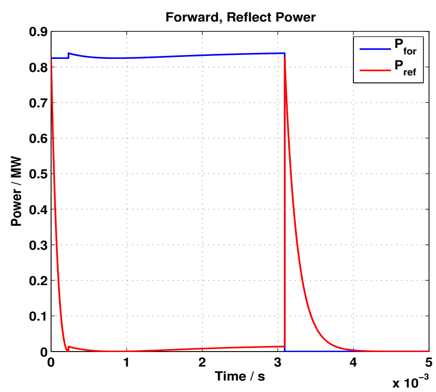

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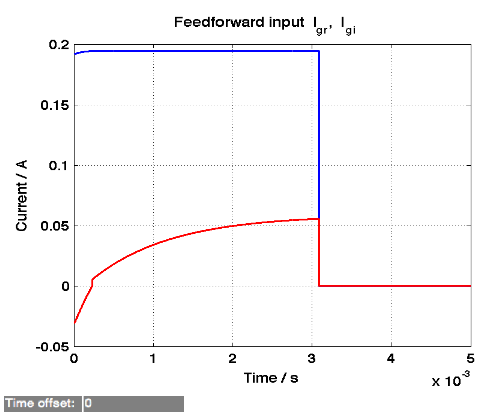

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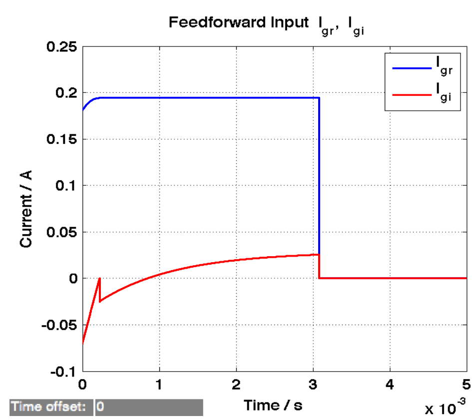
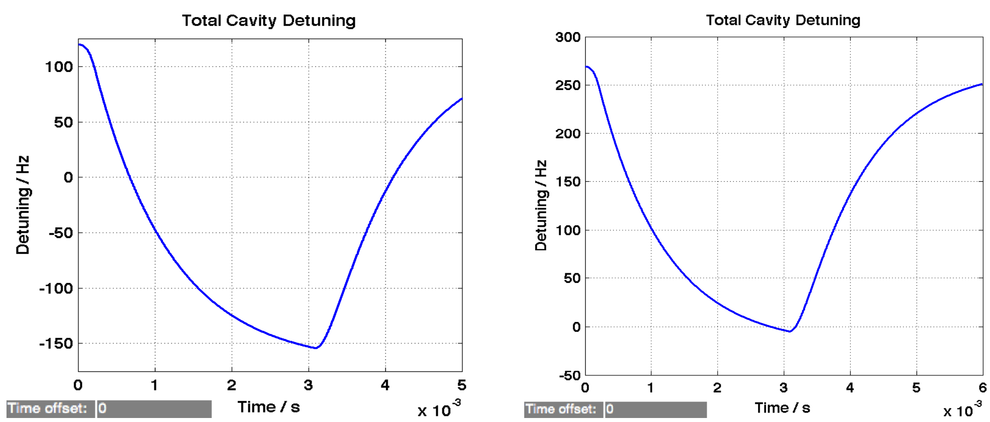


(f)

Figure 3: Cavity field control simulation by feedforward for the cases with (a, c, e) and without $((b, d, f)$ pre-detuning for Lorentz force effect



(a) (b)

Figure 4: Total cavity detuning (lorentz force caused detuning and pre-detuning) for the cases with (a), and without (b) pre-detuning for Lorentz force effect

7 Appendix: a transient analysis for beam injection time

$$\ddot{\mathbf{V}}(t) + \frac{\omega_0}{Q_L}\dot{\mathbf{V}}(t) + \omega_0^2 \mathbf{V}(t) = \frac{\omega_0 R_L}{Q_L}\dot{\mathbf{I}}(\mathbf{t}) \tag{26}$$

$$\mathbf{V}(t) = \mathbf{V}_c(t) \cdot e^{i\omega t} = (V_r(t) + iV_i(t))\, e^{i\omega t} \tag{27}$$

$$\frac{1}{2\omega}\ddot{\mathbf{V}}_\mathbf{c} + i\frac{\omega_0}{2\omega Q_L}\dot{\mathbf{V}}_\mathbf{c} + i\dot{\mathbf{V}}_\mathbf{c} + i\frac{\omega_0}{2Q_L}\mathbf{V}_\mathbf{c} + \frac{1}{2\omega}\left(\omega_0^2 - \omega^2\right)\mathbf{V}_\mathbf{c} = \frac{\omega_0 R_L}{2\omega Q_L}\dot{\mathbf{I}} + i\frac{\omega_0 R_L}{2Q_L}\mathbf{I} \tag{28}$$

For the superconducting cavity, Q_L is higher than 10^5, and the resonance frequency ω_0 closes to the RF frequency ω. Therefore, the first two items in equation left and the first item in the right are negligible, while the item

$\frac{1}{2\omega}\left(\omega_0^2-\omega^2\right) \approx \omega_0-\omega = \Delta\omega$. Then we can get the simplified equation as below:

$$\dot{\mathbf{V}}_{\mathbf{c}} + \frac{\omega_0}{2Q_L}\mathbf{V}_{\mathbf{c}} - i\Delta\omega\mathbf{V}_{\mathbf{c}} = \frac{\omega_0 R_L}{2Q_L}\mathbf{I} \tag{29}$$

Applying the relationship $\omega_{1/2} = \frac{\omega_0}{2Q_L}$ into equation, we have:

$$\dot{\mathbf{V}}_{\mathbf{c}} + \left(\omega_{1/2} - i\Delta\omega\right)\mathbf{V}_{\mathbf{c}} = \omega_{1/2}R_L\mathbf{I} \tag{30}$$

We can easily find out the solution of this first order differential equation as follows:

$$\mathbf{V}_{\mathbf{c}} = \frac{\omega_{1/2}R_L \cdot \mathbf{I}}{\omega_{1/2} - j\Delta\omega}\left(1 - e^{-\left(\omega_{1/2}-j\Delta\omega\right)t}\right) \tag{31}$$

If we immediately cut off the current at the time when the cavity voltage reaches the required value $\mathbf{V}_{\mathbf{req}}$, the expression of the voltage decay curve can be calculated by setting $\mathbf{I} = 0$:

$$\mathbf{V}_{\mathbf{c}} = \mathbf{V}_{\mathbf{req}}\left(e^{-\left(\omega_{1/2}-j\Delta\omega\right)t}\right) \tag{32}$$

The voltage state of the cavity after the beam injection can be viewed as the superposition of the decay of the existed voltage and the newly induced voltages by beam current and generator current which start from the moment of beam injection:

$$\mathbf{V}_{\mathbf{c}} = \mathbf{V}_{\mathbf{req}}\left(e^{-\left(\omega_{1/2}-i\Delta\omega\right)t}\right) + \frac{\omega_{1/2}R_L\left(\mathbf{I}_{\mathbf{g}} - \mathbf{I}_{\mathbf{b}}\right)}{\omega_{1/2} - j\Delta\omega}\left(1 - e^{-\left(\omega_{1/2}-i\Delta\omega\right)t}\right) \tag{33}$$

By combining equation 1 and $tan\varphi_D \approx 2Q_L\frac{\Delta\omega}{\omega_{RF}} = \frac{\Delta\omega}{\omega_{1/2}}$ for the superconducting cavity, we actually can calculate the cavity voltage $\mathbf{V}_{\mathbf{cav}}$ in steady state with the driving current $\mathbf{I}_{\mathbf{total}} = \mathbf{I}_{\mathbf{g}} - \mathbf{I}_{\mathbf{b}}$:

$$\mathbf{V}_{\mathbf{cav}} = \frac{\omega_{1/2}R_L\left(\mathbf{I}_{\mathbf{g}} - \mathbf{I}_{\mathbf{b}}\right)}{\omega_{1/2} - j\Delta\omega} \tag{34}$$

Substituting equation 34 into equation 33, It can be found that the equation $\mathbf{V}_{\mathbf{c}} = \mathbf{V}_{\mathbf{cav}}$ is established as long as the equation $\mathbf{V}_{\mathbf{req}} = \mathbf{V}_{\mathbf{cav}}$ is true. It means the steady state of the cavity voltage will arrive at exact moment of beam injection if we manage to let the required voltage value $\mathbf{V}_{\mathbf{req}}$ (the voltage at the end of filling stage) equal the steady state voltage. It should be kept in mind that $\mathbf{V}_{\mathbf{req}}$ and $\mathbf{V}_{\mathbf{cav}}$ are complex vectors which include both amplitude and phase information.