Steady-state calculations for SC cavities

Stephen Molloy

January 24, 2012

1 Introduction

This document describes the calculations performed by the *rfvectors* Python library to determine the various voltages, powers, etc., for the steady-state solution in a superconducting (SC) RF cavity.

2 The R/Q parameter

As is shown in [1], the coupling of the beam to the cavity field depends on the velocity of the beam, and so it is important to be specific as to the value of the velocity, $v = \beta c$, used when discussing a particular value of the coupling, R/Q.

For the remainder of this note, the coupling with be specified with the associated value of β in subscript. For example, the coupling for a beam moving with the geometric β of the cavity will be shown as, $(R/Q)_{\beta_{geo}}$. In the case where an equation applies generally, and not just to a particular value of β, this will be shown as, $(R/Q)_{\beta}$.

3 Calculations

3.1 Cavity tuning & optimisation

For a cavity with a particular beam coupling, R/Q, through which is accelerated a beam of current, $\mathbf{I_{b0}}$, with an accelerating field of magnitude, $|\mathbf{V_{cav}}|$, and synchronous phase, ϕ_b, the optimum loaded quality factor, Q_{Lopt}, may be shown to be the following.

Note that terms in bold type are complex quantities containing both magnitude and phase information. Throughout the *rfvectors* module, the phase is defined by placing the beam current phasor, $\mathbf{I_{b0}}$, in the negative real direction.

$$Q_{Lopt} = \frac{|\mathbf{V_{cav}}|}{\left(\frac{R}{Q}\right) |\mathbf{I_{b0}}| \cos(\phi_b)} \tag{1}$$

Note that this note distinguishes between the optimimum values of parameters, e.g. Q_{Lopt}, and general values, e.g. Q_L, whose value may not correspond with the optimal.

The optimum fractional detuning of the cavity, $\Delta f/f$, may be shown to be the following.

$$\left(\frac{\Delta f}{f}\right)_{opt} = -\frac{\left(\frac{R}{Q}\right)_\beta |\mathbf{I_{b0}}| \sin(\phi_b)}{2|\mathbf{V_{cav}}|} \tag{2}$$

The tuning angle may be calculated from the fractional frequency.

$$\tan(\psi) = 2Q_L \frac{\Delta f}{f} \tag{3}$$

3.2 Beam induced voltage

The voltage, $\mathbf{V_b}$, induced by the beam current, I_{b0}, may be shown to be the following.

$$\mathbf{V_b} = \cos(\psi)\, e^{i\psi} \left(\frac{R}{Q}\right)_\beta Q_L \mathbf{I_{b0}} \tag{4}$$

3.3 Generator induced voltage

The *rfvectors* module assumes that there is a particular cavity voltage, V_{cav}, that the RF system is designed to produce, and that this must be maintained in the presence of the beam loading given in equation 4.

Note that the cavity voltage, V_{cav}, is defined as the voltage experienced by a particle of a certain velocity, and so includes the transit time factor applicable to that velocity.

$$V_{cav} = E_0 T L = \int_{-L/2}^{L/2} E(x{=}0, y{=}0, z)\, e^{i\frac{\omega z}{\beta c}}\, dz \tag{5}$$

Without beam loading, the cavity would reach a different steady state voltage, $\mathbf{V_g}$, referred to as the generator voltage. The sum of this phasor and that excited by the beam, $\mathbf{V_b}$, should result in the desired cavity voltage.

$$\mathbf{V_g} = \mathbf{V_{cav}} - \mathbf{V_b} \tag{6}$$

This also allows a calculation of the necessary generator power, P_g, to maintain this voltage.

$$P_g = \frac{|\mathbf{V_{cav}}|^2}{\left(\frac{R}{Q}\right)_\beta Q_L} \frac{1}{4} \left[\left(1 + \frac{\left(\frac{R}{Q}\right)_\beta Q_L |\mathbf{I_{b0}}|}{|\mathbf{V_{cav}}|} \cos(\phi_b)\right)^2 + \left(\frac{\Delta f}{f_{1/2}} + \frac{\left(\frac{R}{Q}\right)_\beta Q_L |\mathbf{I_{b0}}|}{|\mathbf{V_{cav}}|} \sin(\phi_b)\right)^2 \right] \tag{7}$$

The beam power, P_b, may be calculated,

$$P_b = |\mathbf{V_{cav}}|\, |\mathbf{I_{b0}}| \cos(\phi_b) \tag{8}$$

Finally, the reflected power, P_{ref}, may be calculated as,

$$P_{ref} = P_g - P_b \tag{9}$$

References

[1] S. Molloy, "Dependence of cavity coupling on beam velocity", Tech Note ESS/AD-147, 2011