Required power for detuned, loaded, SC cavity

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The generator power, P_g, required to maintain a voltage, V_{cav}, in a cavity loaded by a beam current, I_{b0}, depends on the detuning, $\Delta f/f$, of the cavity, the loaded quality factor, Q_L, and the coupling, R/Q, of the beam to the field.

Note that the voltage, V_{cav}, is defined as the electrical field, E_0, that would be experienced by a particle with zero synchronous phase, ϕ_b, integrated over the length, L, of the cavity. The effect of this integral is normally taken care of through the use of the so-called "transit time factor", T.

$$V_{cav} = E_0 \cdot T \cdot L \tag{1}$$

In the case where all parameters are optimised, the power required from the generator to sustain the field can be written in quite a simple form,

$$P_g = \frac{V_{cav}^2}{\left(\frac{R}{Q}\right) Q_L} \tag{2}$$

Equation 2 is the same as the power absorbed by the beam in a well-optimised cavity, and thus results in zero power being reflected from the cavity.

In the general case, however, the detuning of the cavity frequency, and the transformer ratio of the power coupler, won't be perfectly optimised to the accelerating field, beam current, etc., and so the reflected power will be non-zero. In this situation the RF generator will have to provide additional power to sustain the desired field in the cavity, and the expression for this power is as follows.

$$\begin{aligned} P_g = \frac{V_{cav}^2}{\left(\frac{R}{Q}\right) Q_L} \frac{1}{4} \Bigg[& \left(1 + \frac{\left(\frac{R}{Q}\right) Q_L I_{b0}}{V} \cos\left(\phi_b\right)\right)^2 + \\ & \left(\frac{\Delta f}{f_{1/2}} + \frac{\left(\frac{R}{Q}\right) Q_L I_{b0}}{V} \sin\left(\phi_b\right)\right)^2 \Bigg] \end{aligned} \tag{3}$$

Here the cavity bandwidth, $f_{1/2} = f/\left(2Q_L\right)$.

Discussion

The optimal value of the coupling, Q_L, can be shown to be,

$$Q_{Lopt} = \frac{V}{\left(\frac{R}{Q}\right) I_{b0} \cos(\phi_b)} \tag{4}$$

Substituting equation 4 into 3, and using the definition of the cavity bandwidth, $f_{1/2}$, leads to the following,

$$P_g = \frac{V_{cav}^2}{\left(\frac{R}{Q}\right) Q_L} \frac{1}{4} \left[\left(1 + \frac{Q_L}{Q_{Lopt}}\right)^2 + \left(2Q_L \frac{\Delta f}{f} + \frac{Q_L}{Q_{Lopt}} \tan(\phi_b)\right)^2 \right] \tag{5}$$

It can be shown that the optimal value for the detuning is as follows,

$$\frac{\Delta f_{opt}}{f} = -\frac{\tan(\phi_b)}{2Q_L} \tag{6}$$

Substituting equation 6 into 5 leads to the following,

$$P_g = \frac{V_{cav}^2}{\left(\frac{R}{Q}\right) Q_L} \frac{1}{4} \left[\left(1 + \frac{Q_L}{Q_{Lopt}}\right)^2 + \left(2Q_L \frac{\Delta f}{f} \left(1 - \frac{Q_L}{Q_{Lopt}} \frac{\Delta f_{opt}}{\Delta f}\right)\right)^2 \right] \tag{7}$$

Equation 7 shows more clearly that the corrective factor depends on how closely the cavitiy parameters match the optimal set. In particular, it is clear that setting, $Q_L = Q_{Lopt}$, and, $\Delta f = \Delta f_{opt}$ leads to equation 2 as required.

Optimal detuning for a non-optimal coupler.

While it would be prohibitively expensive to alter each coupler in a linac to be correctly optimised, it is relatively simple to adjust the frequency detuning of each of the cavities to minimise the reflected power.

Holding the coupling constant, Q_L, it is possible to differentiate equation 3 to determine the optimal detuning for each cavity.

Defining,

$$f_d \equiv \frac{\Delta f}{f_{1/2}} \tag{8}$$

$$\frac{dP_g}{df_d} = \frac{1}{2} \frac{V_{cav}^2}{\left(\frac{R}{Q}\right) Q_L} \left(f_d + \frac{\left(\frac{R}{Q}\right) Q_L I_{b0}}{V_{cav}} \sin(\phi_b) \right) \tag{9}$$

Setting equation 9 to zero, the optimal detuning, f_{dopt}, is found.

$$f_{dopt} = -\frac{\left(\frac{R}{Q}\right) Q_L I_{b0}}{V} \sin(\phi_b) \tag{10}$$

Substituting the definitions of f_d and $f_{1/2}$, results in the final value for the desired detuning frequency.

$$\Delta f_{opt} = -\frac{1}{2} f \frac{\left(\frac{R}{Q}\right) I_{b0}}{V} \sin(\phi_b) \tag{11}$$

Note that equation 11 has no dependence on Q_L, and so is independent of the design of the coupler. Despite the constant value of Δf, equation 10 shows that the fraction of the bandwidth represented by the detuning does depend on Q_L.

Note that the magnitude of the detuning is typically represented as an angle, Φ, that is defined as follows,

$$\tan(\Psi_{opt}) = f_d = \frac{\Delta f}{f_{1/2}} \tag{12}$$

Thus a detuning of $\Psi > \frac{\pi}{4}$ represents a frequency shift, $\Delta f > f_{1/2}$.

It can be seen from equations 12 and 10 that the optimal detuning angle, Ψ_{opt}, has a strong Q_L dependence.

Detuning for non-optimal beam velocity

In the general case, cavities are used to accelerate beams whose velocity does not match that for which the cavity was designed. In this case it is important to remember that the coupling of the beam to the cavity changes, and that this should be taken into account when substituting a value for the coupling, R/Q, in equations 10 and 11.

Rewriting equation 4 as,

$$Q_{Lopt} = \frac{V}{\left(\frac{R}{Q}\right)_{\beta_0} I_{b0} \cos(\phi_b)} \tag{13}$$

Where the subscript, β_0, shows that the value of the coupling to use here is that for a beam with the design velocity, $v_0 = \beta_0 c$.

Substituting equation 13 into 10, and rewriting the detuning, f_d, as equation 12, leads to the following,

$$\tan(\Psi) = -\frac{\left(\frac{R}{Q}\right)_{\beta}}{\left(\frac{R}{Q}\right)_{\beta_0}} \cdot \frac{Q_L}{Q_{Lopt}} \cdot \tan(\phi_b) \tag{14}$$

Equation 14 shows that, in the case of a cavity accelerating a beam with the design velocity, and with a perfectly matched input coupler, that the detuning angle should be equal to the negative of the synchronous phase of the beam.

In the general case where neither the velocity nor the input coupler are properly matched to the design parameters of the cavity, it shows that the tangent of the detuning angle is scaled from that of the synchronous phase by a value that depends on the mismatches of the beam velocity and the input coupler.