Optimising high-power input couplers

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November 21, 2011

Abstract

Given a free choice of the transformer ratio of the input coupler, and the frequency detuning of the cavity, the power reflected from a beam-loaded RF cavity may be set to zero for a particular accelerating field and beam current. Given that it is unfeasible to choose a different coupling for every cavity in a large linac, the value of the coupling may be chosen so as to minimise the total reflected power from each family of cavities.

This note outlines such a calculation, and confirms that the coupling strengths already chosen for each of the three superconducting cavity families intended to be used in ESS are very close to optimal.

Depending on the commissioning and upgrade scenarios, it may be advantageous to design the couplers for a beam current lower than the nominal machine design, since this may help to increase the range of currents that may be accelerated in this linac.

1 Introduction

For a given cavity, with a particular accelerating voltage phase & magnitude, and a particular beam current loading the fields within this cavity, it is possible to calculate the required forward power from the klystron to maintain this accelerating voltage. This calculation will also provide details of the power reflected from the cavity during this time.

If the designer is entirely free to choose the transformer ratio of the input coupler (and thus the loaded quality factor, Q_L, of the cavity), and the detuning of the resonant frequency, then it is possible to have no reflected power, thus maximising the efficiency of the cavity.

It would be prohibitively expensive to consider designing an optimal input coupler for every cavity in a linac, and so typically only one input coupler will be designed for each family of cavities. This will inevitably lead to reflected power for many of the cavities, and so the problem is then to find the optimum Q_L for each family. In this note, this optimum is defined as the value that leads to the lowest total reflected power for the entire family of cavities operating under nominal conditions.

Typically, the RF parameters of the cavity follow from the beam dynamics requirements, and so are based on achieving a particular cavity voltage, V_{cav}, and phase, ϕ_b, with respect to the beam while being loaded by a given beam current, I_b.

In the following, some basic principles will be given, followed by a discussion of the optimisation procedure, and the consequences for the RF system design for ESS. The derivation of the various formulae, etc., will not be given here, but may be found in [1].

2 Cavity detuning

The amplitude & phase of the voltage excited in the cavity depend on the difference between the frequency of the RF from the power source, ω, and the resonant frequency of the cavity, ω_0. Given the shunt impedance, $R_L = \left(\frac{R}{Q}\right) Q_L$, of the cavity, it can be shown that the phase difference, Ψ, and amplitude, V, of the excited signal are,

$$\tan(\Psi) = Q_L \left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right) \tag{1}$$

$$V = \frac{R_L I}{\sqrt{1 + \tan^2(\Psi)}} \tag{2}$$

Note that the current, I, in equation 2 may be the current of the beam, in which case the voltage is that excited by the passage of the beam, or it may be the current in the coupler, in which case the voltage is that excited by the RF power source. In the case of a beam-induced voltage, it is important to consider the transit-time factor of the cavity [2] when calculating the shunt impedance.

Given a small frequency difference, $\Delta\omega = (\omega_0 - \omega)$, equation 1 may be approximated as,

$$\tan(\Psi) = 2Q_L \frac{\Delta\omega}{\omega} \tag{3}$$

3 Vector calculations

Since the voltages present in a cavity are specified by their amplitude and phase, it is convenient to represent them as having complex amplitude. This allows them to be depicted as vectors in an Argand diagram.

Zero phase will be chosen such that the beam current, $\vec{I_b}$, is in the negative real direction. The cavity voltage will, therefore, be a vector rotated by ϕ_b anti-clockwise from the positive real axis.

Due to the detuning of the cavity, the beam voltage vector, $\vec{V_b}$, will have a phase difference of Ψ from $\vec{I_b}$, and an amplitude given by equation 2.

4 Input voltage

In order to achieve the desired cavity voltage vector, $\vec{V}_{cav}$, the power generator must supply a voltage vector, $\vec{V_g}$, that depends on the beam induced voltage, $\vec{V_b}$.

$$\vec{V}_{cav} = \vec{V_g} + \vec{V_b} \tag{4}$$

The generator voltage, V_g, is excited by applying a "forward voltage", $\vec{V}_{for}$, from the power source. On resonance (i.e. $\Delta\omega = 0$), $\vec{V_g} = 2\vec{V}_{for}$. In general, however, the vector trigonometry leads to a more complex expression,

$$\vec{V}_{for} = \vec{V_g} \frac{\sin(\Psi)}{\sin(\pi - 2\Psi)} \exp(-\Psi i) \tag{5}$$

Here, $i = \sqrt{-1}$, and the complex exponential serves to rotate the vector by the value specified by the detuning of the cavity. This rotation is negative since $\vec{V}_{for}$ is the exciting voltage, and thus the rotation should be positive when moving from $\vec{V}_{for}$ to $\vec{V}_g$.

Given this input voltage, $\vec{V}_{for}$, and the resultant voltage present in the cavity, $\vec{V}_{cav}$, it is then a simple matter to calculate the voltage reflected, $\vec{V}_{ref}$, from the cavity back to the power source.

$$\vec{V}_{cav} = \vec{V}_{for} + \vec{V}_{ref} \tag{6}$$

5 Examples

The previous calculations were done for a series of different cavity parameters, and the results are presented below.

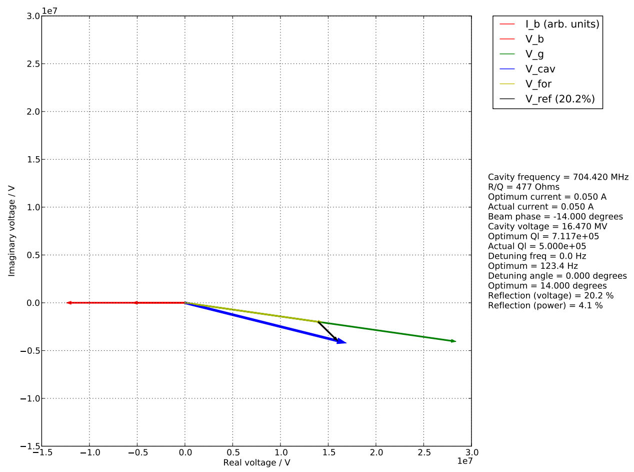

Figure 1: Vector diagram for the various signals within a cavity. No detuning, and a poorly designed input coupler.

Figure 1 shows the signals for a cavity with no detuning (i.e. $\omega_0 = \omega$), and with a poorly optimised input coupler ($Q_L = 5 \times 10^5$, whereas the optimal, $Q_{Lopt} = 7.117 \times 10^5$).

It can be seen that the generator voltage, $\vec{V}_g$, is twice that of the forward voltage, $\vec{V}_{for}$, due to the lack of a detuning angle. In addition, the beam current, $\vec{I}_b$, and the beam induced voltage, $\vec{V}_b$, lie in the same direction.

The reflected power in this case is 4% of the forward power.

Figure 2 shows the signals for a cavity identical to that in figure 1, but with a poorly chosen value for the detuning frequency. It can now be seen that $\vec{I}_b$ and $\vec{V}_b$ are out of phase, as are $\vec{V}_{for}$ and $\vec{V}_g$.

The value of the reflected power for this cavity is 6.3%.

Figure 3 shows the same cavity as the previous figures, but with the detuning set to the optimal frequency. Note from the figure, that this does not result in the optimal detuning angle as this is calculated using the Q_L (see equation 3), which is not set to the optimal value in this calculation.

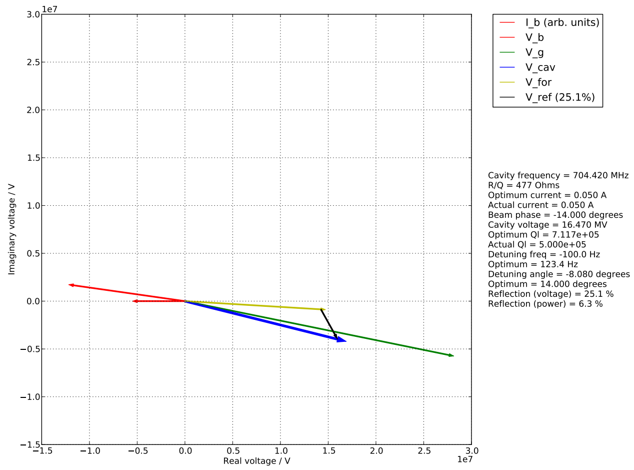

Figure 2: Vector diagram for the various signals within a cavity. Wrong detuning value, and a poorly designed input coupler.

Note that $\vec{V}_{for}$ lies in the same direction as $\vec{V}_{cav}$, thus minimising the reflected power.

Figure 4 shows an optimally designed cavity, with the detuning frequency and input coupler set at their optima. It can be seen that this results in zero reflected power, as expected, and is therefore the most optimal design for power efficiency.

Figure 5 shows the case where the incorrect beam current (a factor of 2 lower than the design) is accelerated in the cavity shown in figure 4. This shows that 11% of the power is reflected in this case, and demonstrates that it is important that the expected range of beam currents is taken into account by the cavity designers. In particular, it should be remembered that the cavities are unlikely to be accelerating full current beam during the commisioning stage of the machine, and that the RF distribution plant has to be designed accordingly.

6 Optimising for a linac

For ESS, there will be over 200 superconducting cavities used to accelerate the beam, and it would be prohibitively expensive to design and build the optimal coupler for each of these cavities. The linac will be split into three “families” of cavities (spokes, low beta ellitpicals, and high beta ellipticals), each of which will have its own coupler design.

Therefore, instead of optimising the coupler to zero the reflected power from each input coupler, the value of Q_L for each coupler family will be chosen to minimise the total reflected power from that section.

The “cavity” class used to perform the calculations shown in section 5 was implemented in Python, and so it was a simple matter to write a function that did the necessary calculations for a linac, and to wrap this into a minimisation routine.

Data files containing the amplitude and phase of $\vec{V}_{cav}$ for each cavity

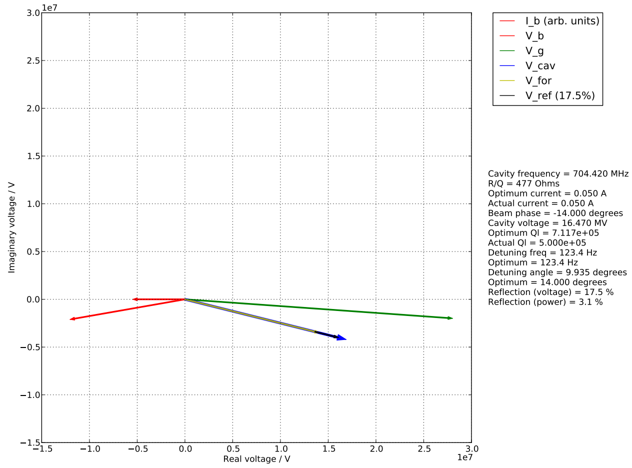

Figure 3: Vector diagram for the various signals within a cavity. Optimal detuning value, and a poorly designed input coupler.

were used to supply the input data for the cavity classes. These files also included the expected velocity, $\beta = v/c$, of the beam as it passed the cavity in order to allow correct scaling of the coupling of the beam to the cavity fields (transit time factor, see [2]).

The Q_L used in each cavity was identical for all cavities in the same family, however it is possible to control the amount of detuning for each cavity, so the calculation was setup to ensure that the frequency detuning was optimal for each cavity.

Figures 6 and 7 show the RF powers for each cavity in the case of the nominal & optimised (respectively) values of Q_L.

The total reflected power for all three cavity families in figures 6 and 7 is 129.3 kW & 123.7 kW, thereby giving only a 5.6 kW power saving.

Due to the fact that this saving is several orders of magnitude smaller than the total forward power (approximately 114 MW), this result may be taken as a confirmation of the suitability of the nominal parameters, rather than suggesting an updated baseline.

Figures 8 and 9 show the consequences of accelerating a beam with a current that differs from that for which the cavities were designed. In this case, they shows the situation that might be observed during the commisioning of the machine, or during an upgrade stage.

In the case of lower than expected current (figure 8), the maximum reflected power reaches approximately 10% of the forward power, while in the case of higher than expected current ((figure 9), the reflected power amounts to only 5% of the input. This implies that, depending on the expected commisioning and upgrade scenarios, it may be advantageous to optimise the couplers for a current lower than that expected in the nominal situation.

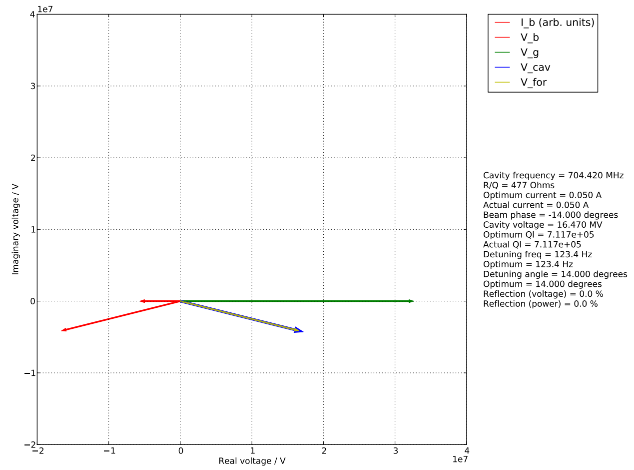

Figure 4: Vector diagram for the various signals within a cavity. Optimal detuning value and input coupler.

7 Conclusions

Code has been written to determine the necessary forward power vector to achieve a particular cavity voltage in the presence of various cavity and beam parameters. In particular, the optimal detuning frequency and input coupling maye be determined, and the effects of operating at non-optimal values for these parameters calculated.

These calculations allow the expected reflected power to be predicted while varying these parameters.

Although it is possible to design and operate a cavity that will have no reflected power for the design beam current, it is not practical to consider designing each cavity as a unique device.

Instead the cavities are grouped into identical families, so the problem then becomes to set the parameters of these families in a way that minimises the total reflected power for the entire linac.

This calculation has been performed, and it has been demostrated that the baseline values for the different couplers are very close to optimal.

Depending on the commissioning and upgrade scenarios, it may be better to consider designing the couplers for a beam current lower than nominal in order to increase the range of currents that may be accelerated in this linac.

References

[1] T. Schilcher, "Vector Sum Control of Pulsed Accelerating Fields in Lorentz Force Detuned Superconducting Cavities", PhD Thesis, Hamburg, 1998

[2] S. Molloy, "Dependence of cavity coupling on beam velocity", ESS Tech Note ESS/AD/0025, 2011

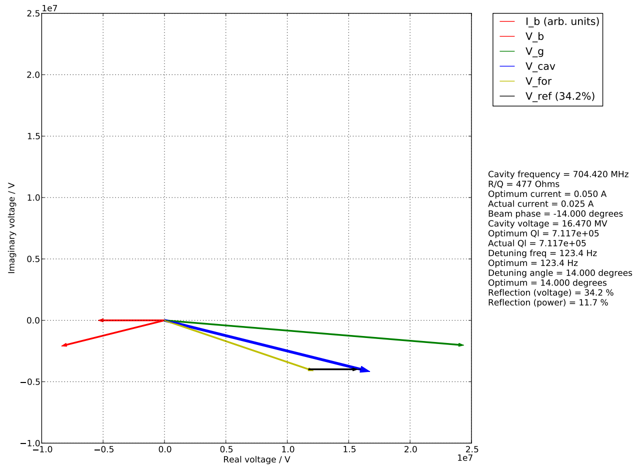

Figure 5: Vector diagram for the various signals within a cavity. Optimal detuning value and input coupler, but with the beam current a factor of 2 lower than the design.

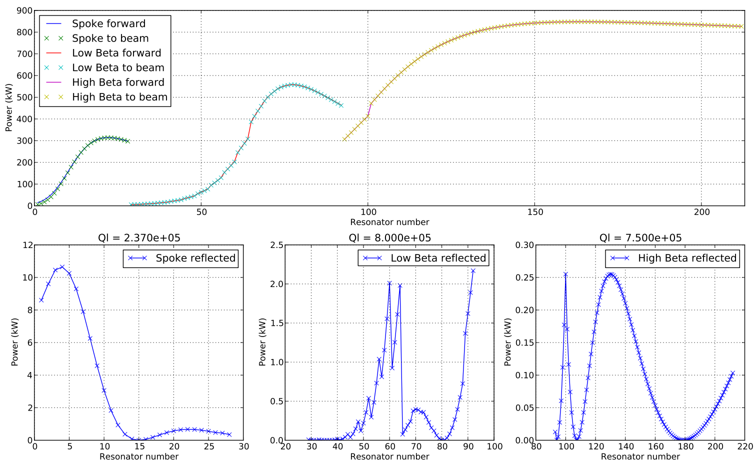

Figure 6: Input & reflected power from each of the superconducting cavities for the nominal values of Q_L.

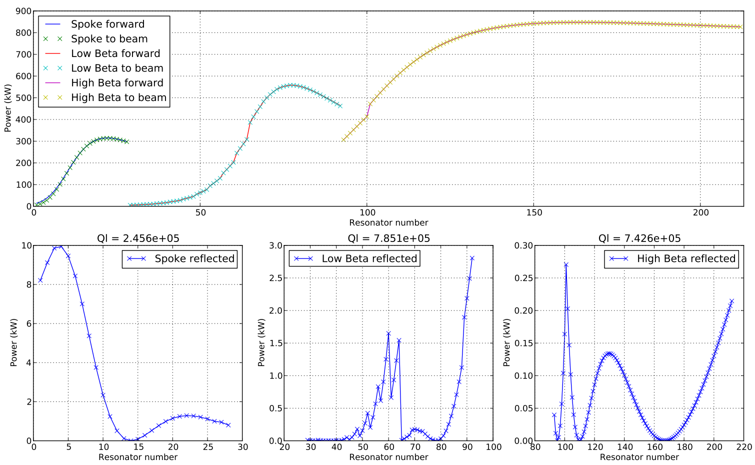

Figure 7: Input & reflected power from each of the superconducting cavities for the optimised values of Q_L.

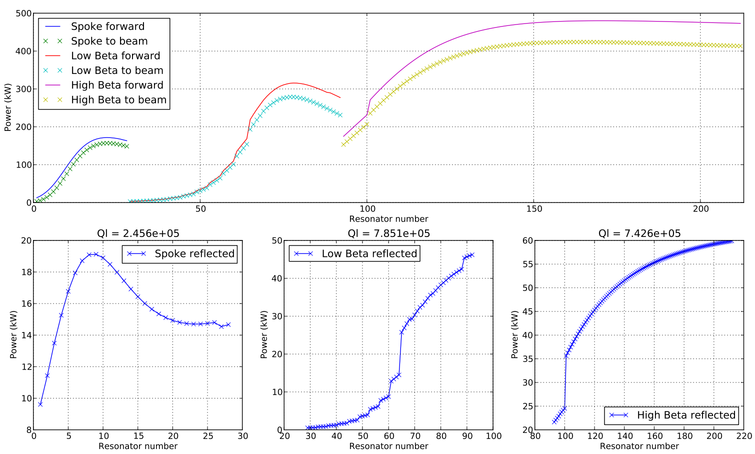

Figure 8: Input & reflected power from each of the superconducting cavities for the nominal values of Q_L, but with a reduction of 50% in the beam current.

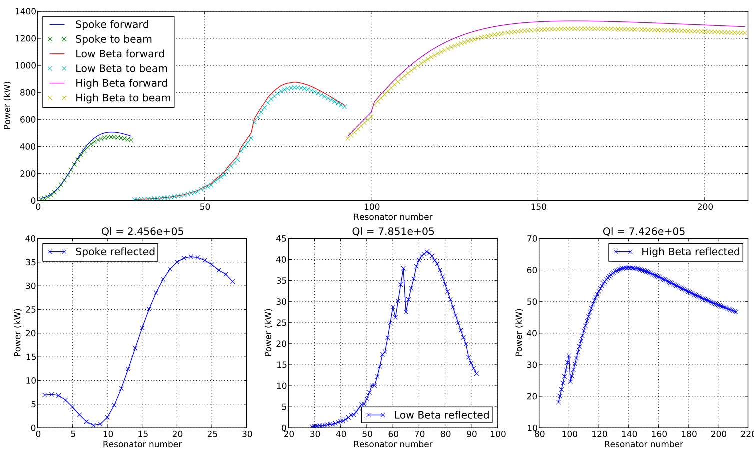

Figure 9: Input & reflected power from each of the superconducting cavities for the optimised values of Q_L, with a 50% increase in the beam current.