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Beam Optics at the ESS Target Interface – For Pedestrians

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Charged-particle optics has many similarities with geometrical optics for light rays. The simplest and most useful lens in particle optics is a quadrupole magnet which 1) keeps the horizontal and vertical motion decoupled and 2) is linear in the sense that the deflection angle of incoming parallel rays is proportional to the distance from the optical axis in each of the two planes. The parallel rays are thus bent towards a point focus. The deflection angle is proportional to the magnetic field strength, and, considering, for instance, the horizontal motion with x and y being the horizontal and vertical coordinates transverse to the beam direction, B_y must then be proportional to x, which indeed is the case for a suitably oriented quadrupole magnet. Similarly, B_x must be proportional to y, which means that a quadrupole magnet is focusing in one plane but defocusing in the other. Otherwise it is very similar to a spherical lens in geometrical optics.

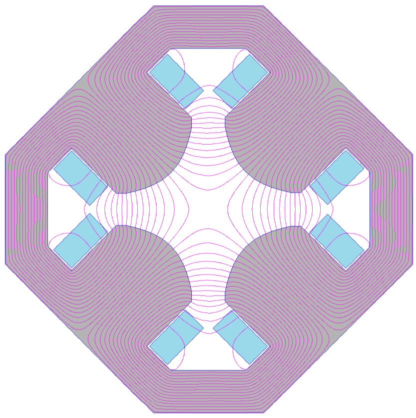

Fig. 1. Cross section of a quadrupole magnet with iron yoke in dark grey, coils in blue and field lines in pink showing that the vertical field component increases linearly with x and that the horizontal field component increases linearly with y.

Using, in addition, s for the coordinate along the beam, it follows from the linearity that the equation of motion of a particle becomes $x'' + K(s)x = 0$, where the derivative is taken with respect to s. Here $K(s)$ is proportional to the quadrupole strength, which is now a function of the longitudinal coordinate all along the machine, depending on where the magnets are located.

It can be shown, although it may not be immediately obvious, that this equation of motion has a solution $x(s) = A\, \beta^{1/2}(s) \cos(\psi(s) + \delta)$. Here, A and δ are two constants of integration, determined by the initial position and direction of motion of the particle, and ψ is the integral of $1/\beta$. In a beam of uncorrelated particles, δ has random values of between 0 and 2π, and the amplitudes A will be distributed up to some maximum value A_{max} that determines the beam size.

The solution shows that the beam particles perform oscillations about the beam axis with an oscillation phase that advances at a varying rate along the linac depending on the magnitude of $\psi(s)$. The beta function only depends on the distribution of magnetic fields and is obtained from $K(s)$ through a differential equation. It can also be calculated step by step through the magnets using linear transfer matrices, like in geometrical optics. In a circular accelerator there is a periodic boundary condition that makes $\beta(s)$ uniquely defined, while in a linac the beta has an undefined scale factor that can be combined with A. This beta function is a central concept in accelerator physics.

Given the maximum oscillation amplitude A_{max}, it is clear from the expression for $x(s)$ that the size of the beam envelope is $A_{max}\ \beta^{1/2}(s)$. For instance, the beam size on the target window is A_{max} times the square root of the beta function at the window, which easily can be calculated according to the above procedure, at least as long as no non-linear elements like octupole magnets are included.

With this background it should be possible to interpret a plot like the one below in Fig. 2. Writing A_{max} as $x_{max}(0)/\beta^{1/2}(0)$ and plotting $\beta^{1/2}(s)/\beta^{1/2}(0)$, it is seen how the beam envelope evolves from some point about 40 m in front of the target up to the target window (or, in target parlance, the proton-beam window). With an assumed initial beam radius of $x_{max}(0)$ = 2 mm, a strong defocusing in front of the target is clearly needed in order to blow the beam up to the provisional 200 mm × 60 mm full width and height of the window.

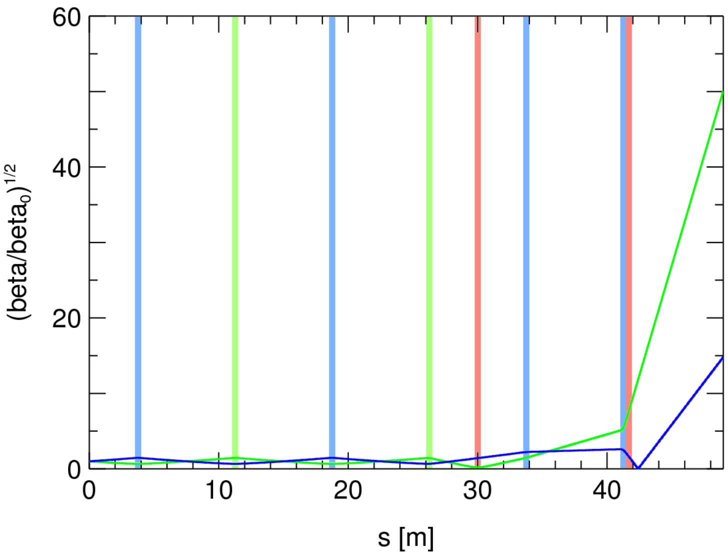

Fig. 2. Beam envelope in the last part of the beam transport line in front of the target. The green line is proportional to the horizontal envelope and the blue line to the vertical envelope. The envelope in mm is obtained by multiplying with the actual beam size at, e.g., x = 0. If the beam radius there is 2 mm × 2 mm, it is seen that it becomes 100 mm × 30 mm at the target. Vertical blue lines indicate the extent of quadrupole lenses that focus horizontally and green are lenses that defocus horizontally. In the vertical plane, the effect of the lenses is the opposite. Read lines represent octupole lenses that do not affect the beam envelope in the linear approximation used for this plot.

In this simple example, the transverse optics of the high-energy beam transport between the linac and the target is taken to be a FODO lattice, i.e., a repeating pattern of a focusing magnet, a drift section, a defocusing magnet and a drift again. All drifts are equally long and at the end there is, as the only deviation from the FODO geometry, a defocusing quadrupole instead of a focusing one.

Continuing with only linear optics, the shape of the intensity distribution across the beam does not change through the beam transport, even if the size of the beam changes with $\beta^{1/2}$ in the two planes. Starting with a Gaussian beam profile (truncated at 3 sigma) at $x = 0$, the distribution is still Gaussian at the position of the target, which is illustrated to the left in Fig. 3.

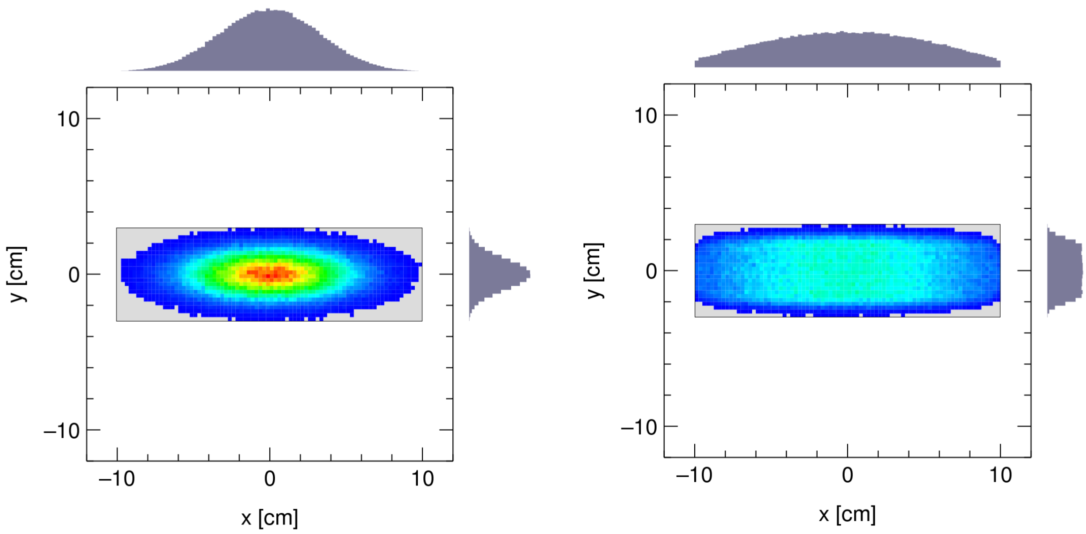

Fig. 3. Beam spot at the position of the target, at the right edge of Fig. 2. To the left is the beam spot and its two projections without octupole magnets, showing Gaussian profiles. At the right the same beam is seen when the beam is more defocused by the quadrupoles and the two octupole fields are added to focus the tails back towards the centre. The areas under the peaks are normalized to the same values in the two plots. The gray rectangle illustrates the assumed extent of the target window.

Adding the octupole magnets changes the beam profile. An octupole magnet has transverse field components that are third-degree polynomials of x and y, and as a result it deflects the tails of the transverse particle distribution stronger than the core. Thus, a more rectangular distribution can be obtained by defocusing more with the quadrupole magnets and at the same time give the outer particles extra focusing in the octupoles, keeping the end of the distribution at the same position. If the octupoles are put where the beam has a waist in one dimension, they will only affect the motion in the other dimension. Two octupoles, one at a location where the beam is small in x and one where it is small in y, as in Fig. 2, makes it possible to control the flattening of the target beam spot more or less indepedently in the two dimensions.

At the right in Fig. 3, the beam profile is seen after the fields in the two octupole magnets have been switched on. The fields are chosen such that the vertical profile is close to rectangular, but with flanks that are not too steep, while the horizontal profile is more peaked towards the centre, which may be beneficial for the neutron yield. This flattening reduces the peak intensity (particles per cm^{-2}) in the right right plot to 35% of that in the left plot.

A realistic HEBT must, in contrast to the one of Fig. 2, have one or two bends to stop backstreaming neutrons and perhaps also to bring the proton beam up from the linac tunnel to the level of the target. In order to further optimize the beam profile at the target, more than the two degrees of freedom used here—the currents in the two octupole magnets—may also be required, possibly in the form of higher-order multipole magnets such as duodecapoles. The sensitivity to errors such as variations in beam parameters and magnet currents must be examined, and it should not be excluded to scan the beam across the target rather than defocus it as in this note.